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ITISE 2022

June 30, 2022

Comparison of different portfolio optimization strategies to minimize the risk

Aleš Kresta, Anlan Wang

VSB – Technical University of Ostrava

Faculty of Economics, Department of Finance

Outline

- Background and Significance of the Research
- Objective
- Methodologies of Portfolio Optimization
- Methodologies of Portfolio Historical Performance Verification
- Empirical Studies on Portfolio Historical Performance Verification
- Conclusion

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Background and Significance of the Research

- **Portfolio Strategies**
- Generally, the strategies made by the portfolio managers are expected to generate higher returns than the random investments
- However, Malkiel (1973) stated an opposite opinion, that is

Background and Significance of the Research

? So, we may wonder whether the portfolio strategies provided by models perform better than the random portfolio investments

- **Developments of Portfolio Optimization Models**
- Markowitz (1952): mean-variance model, had laid the foundation of the research of portfolio optimization.
- With the developments of financial theories and mathematical methods, some modern models have also been proposed, e.g., the model maximizing the portfolio Rachev ratio (Rachev et al., 2005).

- **Estimation Errors**
- However, the optimization models depend on the estimation of the future returns' probability distribution may be subject to errors, see e.g., DeMiguel et al. (2009).
- Thus, the approaches of portfolio weights determination may not provide an advantage compared to just randomly selected portfolio composition.

Background and Significance of the Research

- **Martinez-Nieto et al. (2021)**
- The portfolio performances of 11 selected diversification strategies were compared.

- **Ledoit and Wolf (2008, 2011)**
- A benchmark was applied in the performance verification of portfolio strategies.
- In the meanwhile, they also applied the bootstrapping method to make the performance hypothesis testing by means of Sharpe ratio (Ledoit and Wolf, 2008) or the variance of returns (Ledoit and Wolf, 2011).

Background and Significance of the Research

➢ ***Research Gap 1.***

- In relevant papers, the portfolio performance verifications were made focused on the application of the Sharpe ratio, which is a measure of the relationship between the excess return relative to risk-free return and its standard deviation, see Sharpe (1966).
- However, just as discussed by Eling and Schuhmacher (2007), **Sharpe ratio is not always an adequate performance measure, especially when the investor splits the risky assets or when the returns are not normally distributed.**

➢ ***Research Gap 2.***

- **Comparing the performance of random portfolios with that of strategy portfolios has practical significance, however, at present, there are only few relevant literature discussing about this type of comparison.**

Objective

✓ **The objective is to make the verification on the historical performance of strategy portfolios obtained by applying the classical and modern portfolio optimization models.**

- More specifically,

➢ To make the verification, similarly as in the research of Kim and Lee (2016), we compare the performance of the strategy portfolios with that of the uniformly distributed random portfolios.

➢ However, in our approach, we do not provide the closed-form expression for the distribution of the Sharpe ratio, but rather depend on applying the Monte Carlo simulation method to generate the random portfolios, by this way, it allows us to consider also other performance and risk measures and not be limited only to the Sharpe ratio.

Methodologies of Portfolio Optimization

- **Application of Risk Measures**

Table: Applied Risk Measures

Abbreviation	**Risk Measure**
STD	annualized standard deviation of portfolio returns
MAD	annualized mean absolute deviation of portfolio returns
CVAR1	conditional value at risk for α=1%
CVAR5	conditional value at risk for α=5%
CVAR10	conditional value at risk for α=10%
ML	maximum observed one-day loss
LPM	lower partial moment for u=2
MAXDD	maximum drawdown

Methodologies of Portfolio Optimization

- **Application of Performance Measures**

- For the Sortino-Satchell Ratio, Farinelli-Tibiletti ratio and Rachev ratio, we set 3, 5 and 4 pairs of commonly recommended values to the parameters of them, respectively

Table: Applied Performance Measures

Abbreviation	Performance Measure
MR	annualized mean return
JA	Jensen's Alpha
TR	Treynor ratio
SR	Sharpe ratio
SS08	Sortino-Satchell ratio for u=0.8
SS2	Sortino-Satchell ratio for u=2
SS25	Sortino-Satchell ratio for u=2.5
FT05	Farinelli-Tibiletti ratio for γ=0.5, δ=2
FT2	Farinelli-Tibiletti ratio for γ=2, δ=0.5
OMG	Farinelli-Tibiletti ratio for $\gamma=\delta$=1 (i.e., Omega index)
FT10	Farinelli-Tibiletti ratio for γ=10, δ=0.01
FT001	Farinelli-Tibiletti ratio for γ=0.01, δ=10
RR5	Rachev ratio for $\alpha=\beta$=0.05
RR50	Rachev ratio for $\alpha=\beta$=0.5
RR75	Rachev ratio for $\alpha=\beta$=0.75
STARR5	Rachev ratio for α=0.05, β=1 (i.e., STARR (5%) ratio)
CR	Calmar ratio

Methodologies of Portfolio Optimization

- **Application of Portfolio Optimization Models**

- The first letter of each abbreviation indicates we minimize ("N") the corresponding risk measure of the portfolio
- The number at the end of the abbreviation indicates the setting value to parameter of the corresponding risk measure

Table: Applied Minimum-Risk Portfolio Optimization Models

Abbreviation	**Portfolio Optimization Model**
NVHS	minimum variance – *Historical Sample* estimation
NVBS	minimum variance – *Bayes-Stein* estimation
NVFU	minimum variance – *Fuzzy* estimation
NMAD	minimum MAD
NCVAR1	minimum CVaR for α=1%
NCVAR5	minimum CVaR for α=5%
NCVAR10	minimum CVaR for α=10%
NSV	minimum semivariance

Methodologies of Portfolio Optimization

- **Application of Portfolio Optimization Models**

- The first letter of each abbreviation indicates we maximize ("X") the corresponding performance measure of the portfolio
- The number at the end of the abbreviation indicates the setting values to parameters of the corresponding performance measure

Table: Applied Maximum-Performance Portfolio Optimization Models

Abbreviation	**Portfolio Optimization Model**
XRHS	maximum return – *Historical Sample* estimation
XRFU	maximum return – *Fuzzy* estimation
XSR	maximum Sharpe ratio
XRR5	maximum Rachev ratio for $\alpha=\beta=0.05$
XRR50	maximum Rachev ratio for $\alpha=\beta=0.5$
XRR75	maximum Rachev ratio $\alpha=\beta=0.75$
XSTARR5	maximum STARR ratio for $\alpha=0.05$
XSS08	maximum Sortino-Satchell ratio for $u=0.8$
XSS2	maximum Sortino-Satchell ratio for $u=2$
XSS25	maximum Sortino-Satchell ratio for $u=2.5$
XFT05	maximum Farinelli-Tibiletti ratio for $\gamma=0.5$, $\delta=2$
XFT2	maximum Farinelli-Tibiletti ratio for $\gamma=2$, $\delta=0.5$
XOMG	maximum Omega index
XFT10	maximum Farinelli-Tibiletti ratio for $\gamma=10$, $\delta=0.01$
XFT001	maximum Farinelli-Tibiletti ratio for $\gamma=0.01$, $\delta=10$
NAIVE	naive
EWRC	equally weighted risk contribution

Methodologies of Portfolio Historical Performance Evaluation

- **Monte Carlo Simulation Approach**
- Procedures:

1) choose $y \in [0,1]^{N-1}$ as *N*-1 real numbers uniformly distributed in the interval [0,1]
2) sort these numbers so that $0 \leq y_1 \leq \cdots \leq y_{N-1}$
3) after hypothetically adding the values zero and one to these numbers, we obtain the random weights w as the differences of the successive pairs of numbers in y,
$$w = (y_1, y_2 - y_1, y_3 - y_2, \dots, y_{N-1} - y_{N-2}, 1 - y_{N-1})$$
where the sum of w equals to 1 in each generated portfolio

- Two examples:
- For 4-assets portfolio and 16-assets portfolio, we both generate 100,000 random portfolios

Methodologies of Portfolio Historical Performance Verification

Table: Characteristics of the Simulated Random Weights Distributions for 4-Assets Portfolio

Characteristics		Asset A	Asset B	Asset C	Asset D	Beta (1;3)
Mean		24.95%	24.99%	25.00%	25.06%	25.00%
Standard Deviation		19.41%	19.37%	19.36%	19.35%	19.36%
Skewness		0.87	0.86	0.86	0.86	0.86
Kurtosis		3.11	3.11	3.09	3.10	3.10
Percentile	1%	0.35%	0.32%	0.32%	0.35%	0.33%
	5%	1.68%	1.70%	1.67%	1.72%	1.70%
	10%	3.41%	3.43%	3.45%	3.52%	3.45%
	25%	9.03%	9.10%	9.13%	9.19%	9.14%
	50%	20.54%	20.65%	20.62%	20.73%	20.63%
	75%	36.98%	36.98%	37.04%	36.90%	37.00%
	90%	53.55%	53.57%	53.50%	53.77%	53.58%
	95%	63.36%	63.14%	63.15%	63.32%	63.16%
	99%	78.63%	78.41%	78.37%	78.43%	78.46%

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Table: Characteristics of the Simulated Random Weights Distributions for 16-Assets Portfolio

Characteristics		**Asset A**	**Asset B**	**Asset C**	**Asset D**	**Beta (1;15)**
Mean		6.28%	6.28%	6.24%	6.26%	6.25%
Standard Deviation		5.88%	5.90%	5.87%	5.88%	5.87%
Skewness		1.64	1.63	1.66	1.66	1.66
Kurtosis		6.43	6.38	6.60	6.64	6.58
Percentile	1%	0.06%	0.07%	0.07%	0.07%	0.07%
	5%	0.34%	0.34%	0.35%	0.34%	0.34%
	10%	0.71%	0.69%	0.70%	0.70%	0.70%
	25%	1.92%	1.89%	1.89%	1.89%	1.90%
	50%	4.56%	4.52%	4.52%	4.54%	4.52%
	75%	8.87%	8.89%	8.78%	8.85%	8.83%
	90%	14.27%	14.33%	14.17%	14.23%	14.23%
	95%	18.15%	18.19%	18.13%	18.03%	18.10%
	99%	26.52%	26.41%	26.51%	26.59%	26.44%

Methodologies of Portfolio Historical Performance Verification

- **Single-Period Testing**
- It's a method applied for testing the single-period out-of-sample results, in particular, the testing can be further designed for the minimum-risk portfolios and the maximum-performance portfolios, respectively
- Hypothesis testing:

H_0: *The value of the risk (performance) measure of the strategy portfolio is neither lower nor higher than the median value of the risk (performance) measure of randomly-generated portfolios.*

H_1: *The value of the risk (performance) measure of the strategy portfolio is lower (higher) than the median value of the risk (performance) measure of randomly-generated portfolios.*

- Furthermore, the null hypothesis can be refined based on the concept of **"relative ranking".** The relative ranking is always obtained on the series of values of the risk (performance) measure of random portfolios sorted from the lowest value to the highest value. Note that for a risk measure, the lowest value is the best; while for a performance measure, the highest value is the best.

Methodologies of Portfolio Historical Performance Verification

- Hypothesis testing:
- Considering the risk measures,

$$H_0\text{: } \textit{The mean relative ranking of the strategy portfolio is equal to 50\%.}$$

$$H_1\text{: } \textit{The mean relative ranking of the strategy portfolio is lower than 50\%.}$$

- Considering the performance measures,

$$H_0\text{: } \textit{The mean relative ranking of the strategy portfolio is equal to 50\%.}$$

$$H_1\text{: } \textit{The mean relative ranking of the strategy portfolio is higher than 50\%.}$$

- Since we compare the mean of a single population with a hypothesized mean (50%), so, the One-Sample Z-test can be applied in in our case.

Empirical Studies on Portfolio Historical Performance Verification

- **Data Description**
- To verify the strategy portfolios' historical performance, we make three empirical case studies

1) The applied data of the three case studies are the historical daily adjusted closing prices of stocks included in Dow Jones Industrial Average (DJIA) index. The applied datasets in these three case studies are different in the periods of analysis

2) In order to avoid the survivorship bias, we take the composition of DJIA as of January 3rd, 2006 (i.e., the beginning of evaluation periods of the case studies)

3) Among the 30 components of DJIA, due to the availability of data, in case study 1 and case study 3, we exclude the stocks of *General Motors Corporation*, *Hewlett-Packard Company* and *United Technologies Corporation*

Empirical Studies on Portfolio Historical Performance Verification

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Table: Component Stocks of DJIA

Ticker Symbol	**Company Name**
MMM	3M Company
AXP	American Express Company
BA	The Boeing Company
CAT	Caterpillar Inc.
KO	The Coca-Cola Company
HD	The Home Depot, Inc.
INTC	Intel Corporation
IBM	International Business Machines Corporation
JNJ	Johnson & Johnson
JPM	JPMorgan Chase & Co.
MCD	McDonald's Corporation
MRK	Merck & Co., Inc.
MSFT	Microsoft Corporation
PG	The Procter & Gamble Company
VZ	Verizon Communications Inc.

Ticker Symbol	**Company Name**
WMT	Wal-Mart Stores, Inc.
DIS	The Walt Disney Company
AA	Alcoa Inc.
MO	Altria Group Incorporated
AIG	American International Group, Inc.
C	Citigroup Inc.
T	AT&T Inc.
DD	E.I. du Pont de Nemours & Company
XOM	Exxon Mobil Corporation
GE	General Electric Company
HON	Honeywell International Inc.
PFE	Pfizer Inc.
GM	General Motors Corporation
HPE	Hewlett-Packard Company
UTX	United Technologies Corporation

Empirical Studies on Portfolio Historical Performance Verification – Case Study 1

- **Case Study 1 – 2007 – 2009 Global Financial Crisis**
- The analyzed period of case study 1 covers the change in the market trend due to the 2007 – 2009 Global Financial Crisis (GFC), at this background, we can especially verify the success of the minimum-risk models

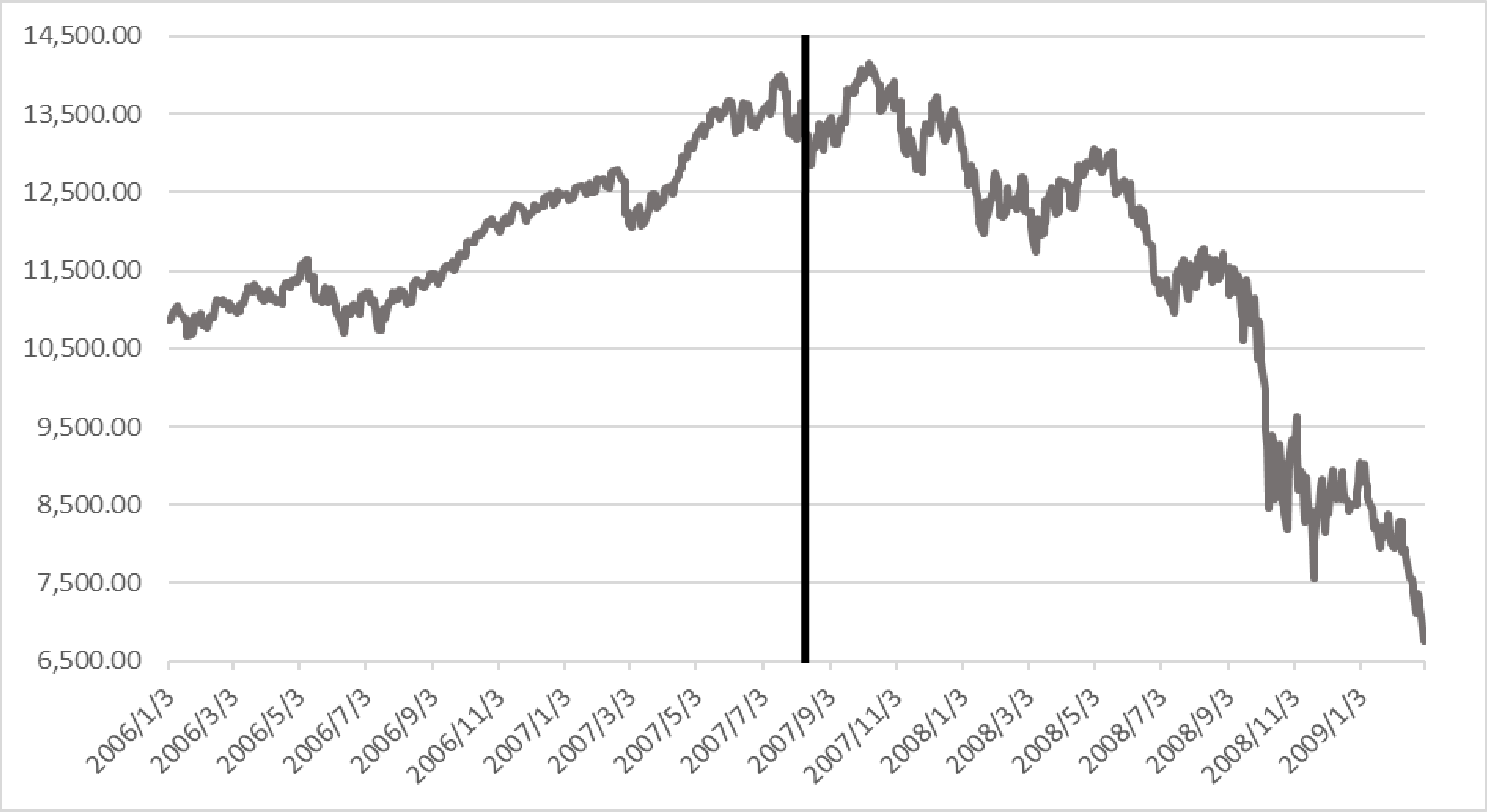
- In-sample period: January 3, 2006 – August 10, 2007

  – 404 in-sample observations
- Out-of-sample period: August 13, 2007 – March 2, 2009

  – 391 out-of-sample observations

Empirical Studies on Portfolio Historical Performance Verification – Case Study 1

Figure: DJIA Value Evolution During 2007–2009 GFC
(In-Sample and Out-of-Sample Period)



Source: Own representation of data from www.finance.yahoo.com

Empirical Studies on Portfolio Historical Performance Verification – Case Study 1

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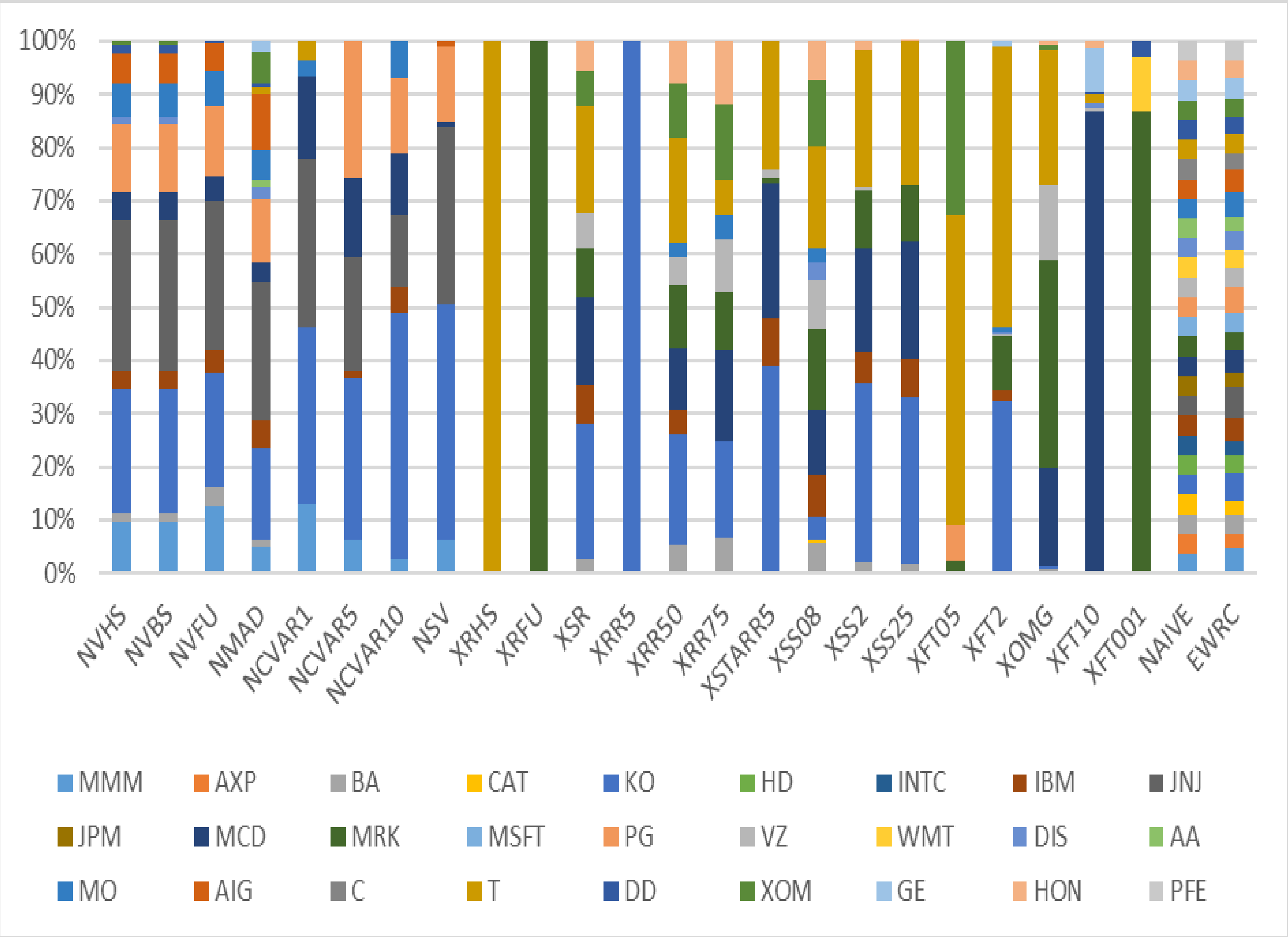
Figure: Compositions of Strategy Portfolios – Case 1



- Minimum-Variance strategy portfolios (NVHS, NVBS and NVFU) have similar compositions
- For portfolios obtained from the same model but with different values to the parameters, they have similar components but are different in the weights (i.e., NCVAR5 vs. NCVAR10, XRR50 vs. XRR75, XSS2 vs. XSS25)
- There are single-stock portfolios suggested by Maximum-Return strategies (XRHS and XRFU) and Maximum-Rachev strategy for $\alpha = \beta = 0.05$ (XRR5).

Table: Acceptances and Rejections of Null Hypothesis at 5% significance level – Case 1

	STD	MAD	CVAR1	CVAR5	CVAR10	ML	LPM	MAXDD	MR	JA	TR	SR	SS08	SS2	SS25	FT05	FT2	OMG	FT10	FT001	RR5	RR50	RR75	STARR5	CR
NVHS																									
NVBS																									
NVFU																									
NMAD																									
NCVAR1																									
NCVAR5																									
NCVAR10																									
NSV																									
XRHS																									
XRFU																									
XSR																									
XRR5																									
XRR50																									
XRR75																									
XSTARR5																									
XSS08																									
XSS2																									
XSS25																									
XFT05																									
XFT2																									
XOMG																									
XFT10																									
XFT1																									
NAIVE																									
EWCR																									

Note: the strategy portfolios for which we should reject the null hypothesis are colored in green and for which we cannot reject the null hypothesis are colored in red.

- Strategies minimizing the in-sample variance lower the risk in the out-of-sample period. While they perform randomly in terms of the performance measures.
- Strategies minimizing the in-sample CVaR and semivariance, they performed very well concerning most of the applied risk measures and performance measures
- Equally weighted strategies perform poorly in the out-of-sample period.

Empirical Studies on Portfolio Historical Performance Verification – Case Study 2

- **Case Study 2 – Covid-19 Pandemic**
- In order to make our verifications more reliable, we also make another case study which covers the stock market crisis due to the Covid-19 Pandemic

- In-sample period: March 6, 2018 – October 10, 2019

  – 404 in-sample observations
- Out-of-sample period: October 11, 2019 – April 30, 2021

  – 391 out-of-sample observations

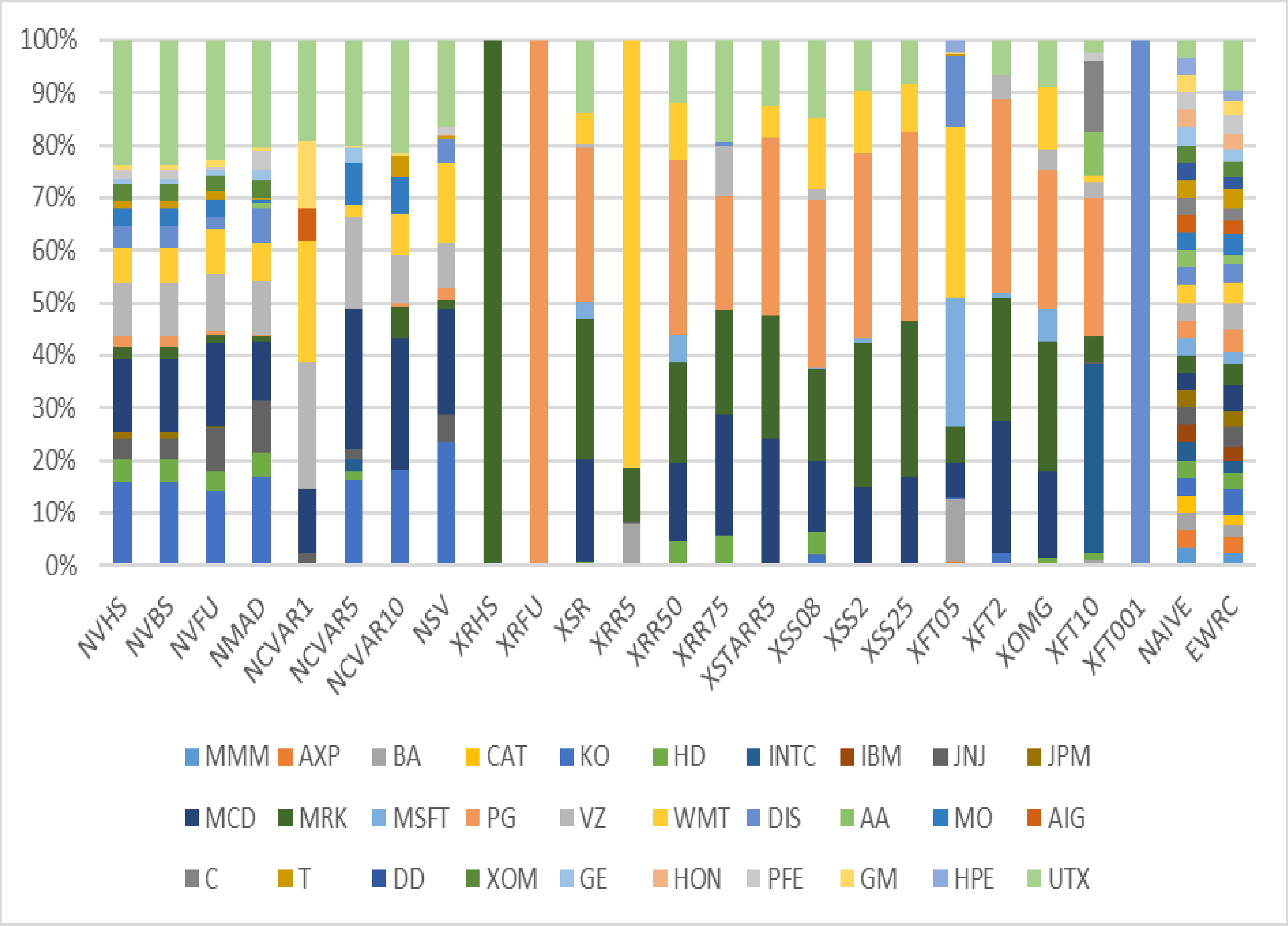
Empirical Studies on Portfolio Historical Performance Verification – Case Study 2

Figure: DJIA Value Evolution During Covid-19 Pandemic
(In-Sample and Out-of-Sample Period)

Source: Own representation of data from www.finance.yahoo.com

Empirical Studies on Portfolio Historical Performance Verification – Case Study 2

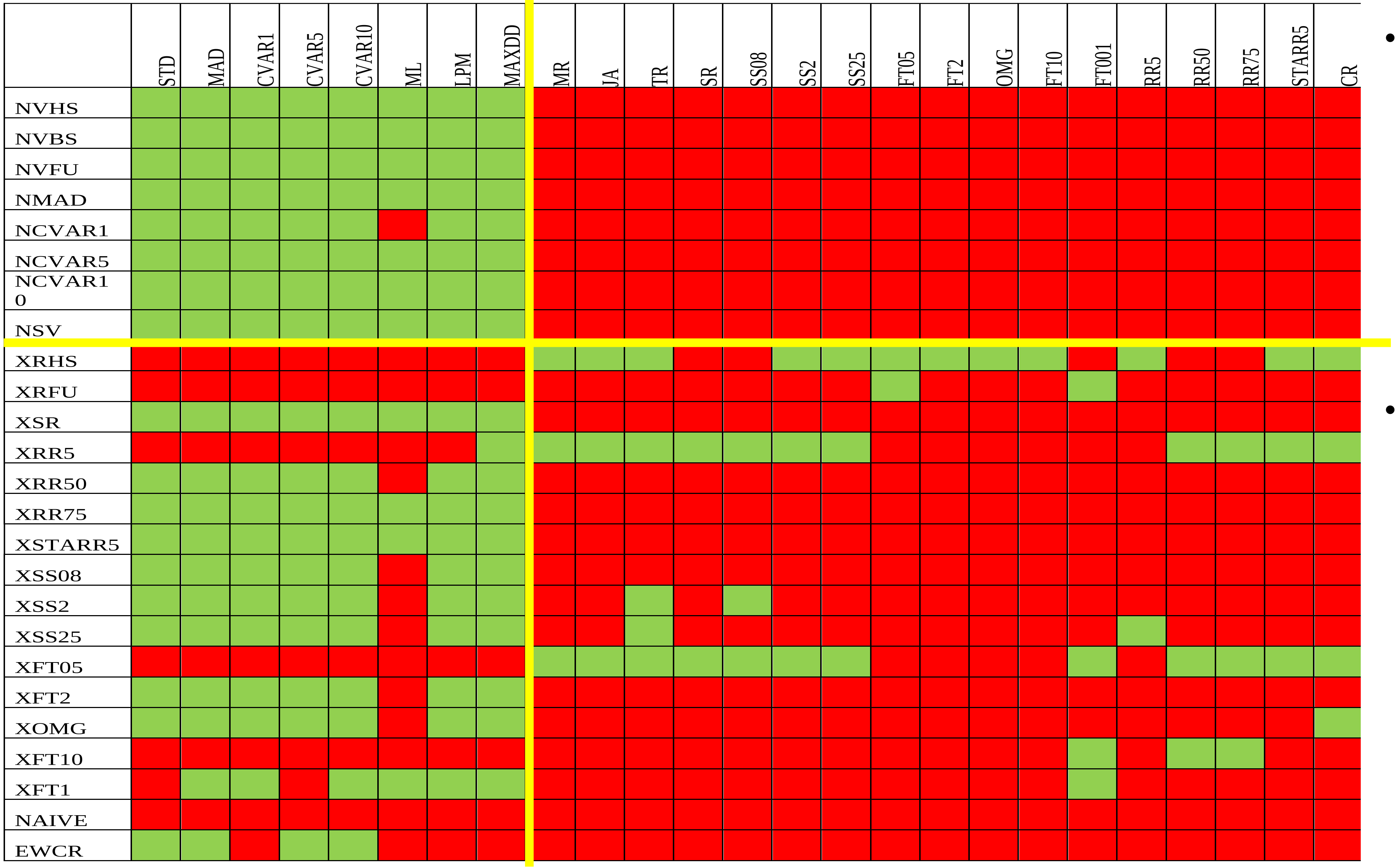
Figure: Compositions of Strategy Portfolios – Case 2



- Minimum-Variance (NVHS, NVBS and NVFU) and the Minimum-MAD (NMAD) strategies suggest similar portfolio compositions.

- There are single-stock portfolios suggested by Maximum-Return strategies (XRHS and XRFU) and Maximum Farinelli-Tibiletti strategy for $\gamma = 0.01, \delta = 10$ (XFT001).

Table: Acceptances and Rejections of Null Hypothesis at 5% significance level – Case 2



- The strategies minimizing in-sample risk (except for NCVAR1) perform well in minimizing the out-of-sample risk, however, they perform randomly considering all the applied performance measures

- In general, the strategies which can be verified as improved the out-of-sample performance measures are XRHS, XRR5 and XFT05

Note: the strategy portfolios for which we should reject the null hypothesis are colored in green and for which we cannot reject the null hypothesis are colored in red.

Empirical Studies on Portfolio Historical Performance Verification – Case Study 3

- **Case Study 3 – Rolling Window Analysis**

- In this case study, we verify the historical performance of strategy portfolios concerning the changes of periods.

- So, in this case, we apply the rolling window analysis and make multi-period testing of the obtained multiple out-of-sample results.

- By this way, we can increase the robustness of the obtained results.

Empirical Studies on Portfolio Historical Performance Verification – Case Study 3

- The applied datasets in this case study cover the period from January 3, 2006 to April 30, 2021.
- In particular, we always take 1000 days as the in-sample period and 250 days as the out-of-sample period, then we move the start of the evaluation window day by day.
- In total, we have 2,607 overlapping 250-days out-of-sample periods for which the investments are made.

In-Sample Periods	Out-of-Sample Periods
January 3, 2006 — December 21, 2009	December 22, 2009 — December 17, 2010
January 4, 2006 — December 22, 2009	December 23, 2009 — December 18, 2010
…	…
May 13, 2016 — May 4, 2020	May 5, 2020 — April 30, 2021

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Table: Mean Relative Ranking of Strategy Portfolios for Risk Measures

	STD	MAD	CVAR1	CVAR5	CVAR10	ML	LPM	MAXDD
NVHS	**0.2%**	**0.7%**	**4.5%**	**1.0%**	**0.9%**	**14.8%**	**1.0%**	**28.1%**
NVBS	**0.2%**	**0.7%**	**4.5%**	**1.0%**	**0.9%**	**14.8%**	**1.0%**	**28.1%**
NVFU	**0.3%**	**1.1%**	**5.0%**	**1.8%**	**1.8%**	**16.3%**	**1.6%**	**26.9%**
NMAD	**0.1%**	**0.2%**	**4.3%**	**1.3%**	**0.9%**	**15.2%**	**1.1%**	**28.7%**
NCVAR1	**4.9%**	**6.5%**	**8.8%**	**3.1%**	**4.2%**	**17.6%**	**5.8%**	**27.7%**
NCVAR5	**2.0%**	**5.1%**	**6.8%**	**0.9%**	**2.1%**	**17.0%**	**1.5%**	**30.1%**
NCVAR10	**1.2%**	**3.2%**	**8.5%**	**1.5%**	**2.0%**	**18.5%**	**2.4%**	**30.8%**
XRHS	89.1%	89.4%	86.6%	87.0%	87.8%	83.4%	89.7%	70.4%
XRFU	97.1%	97.3%	94.5%	95.0%	95.7%	91.6%	97.3%	80.4%
XSR	53.3%	53.5%	52.7%	**47.6%**	**46.2%**	55.6%	53.1%	**46.9%**
XRR5	90.7%	90.9%	85.8%	87.8%	87.3%	81.1%	90.4%	78.0%
XRR50	**48.6%**	**48.6%**	**48.3%**	**42.7%**	**41.7%**	52.5%	**48.5%**	**44.4%**
XRR75	51.6%	50.1%	49.2%	**47.4%**	**45.0%**	51.2%	49.5%	51.0%
XSTARR5	53.5%	55.0%	52.2%	**47.2%**	**46.4%**	56.6%	53.0%	**47.5%**
XSS08	**44.7%**	**44.3%**	**46.3%**	**40.8%**	**39.8%**	50.5%	**45.7%**	**42.1%**
XSS2	52.4%	53.1%	50.3%	**46.2%**	**45.6%**	55.6%	51.6%	**43.7%**
XSS25	54.0%	54.8%	51.2%	**48.2%**	**47.7%**	57.1%	53.5%	**44.0%**
XFT05	88.3%	88.1%	83.7%	86.5%	86.2%	81.1%	88.0%	61.9%
XFT2	59.5%	60.8%	57.2%	57.0%	55.3%	55.3%	59.6%	55.7%
XOMG	**47.2%**	**47.7%**	**46.2%**	**43.2%**	**42.1%**	52.5%	**46.7%**	**39.9%**
XFT10	87.7%	88.8%	82.2%	85.4%	85.2%	76.1%	86.1%	80.8%
XFT001	87.9%	88.6%	84.7%	84.9%	85.5%	84.6%	86.1%	80.7%
NAIVE	**36.3%**	**33.4%**	**45.1%**	**41.0%**	**39.0%**	**44.8%**	**41.1%**	**42.8%**

- In **bold** we highlight the mean relative rankings which indicate the strategies lowered the risk measures
- Naive portfolio performs better than random portfolios in lowering the risk
- The strategies minimizing the Variance, MAD and CVaR, their risk is lowered in the out-of-sample periods for all risk measures
- Low out-of-sample risk can also be observed for strategies XRR50, XSS08, and XOMG

Table: Mean Relative Ranking of Strategy Portfolios for Performance Measures

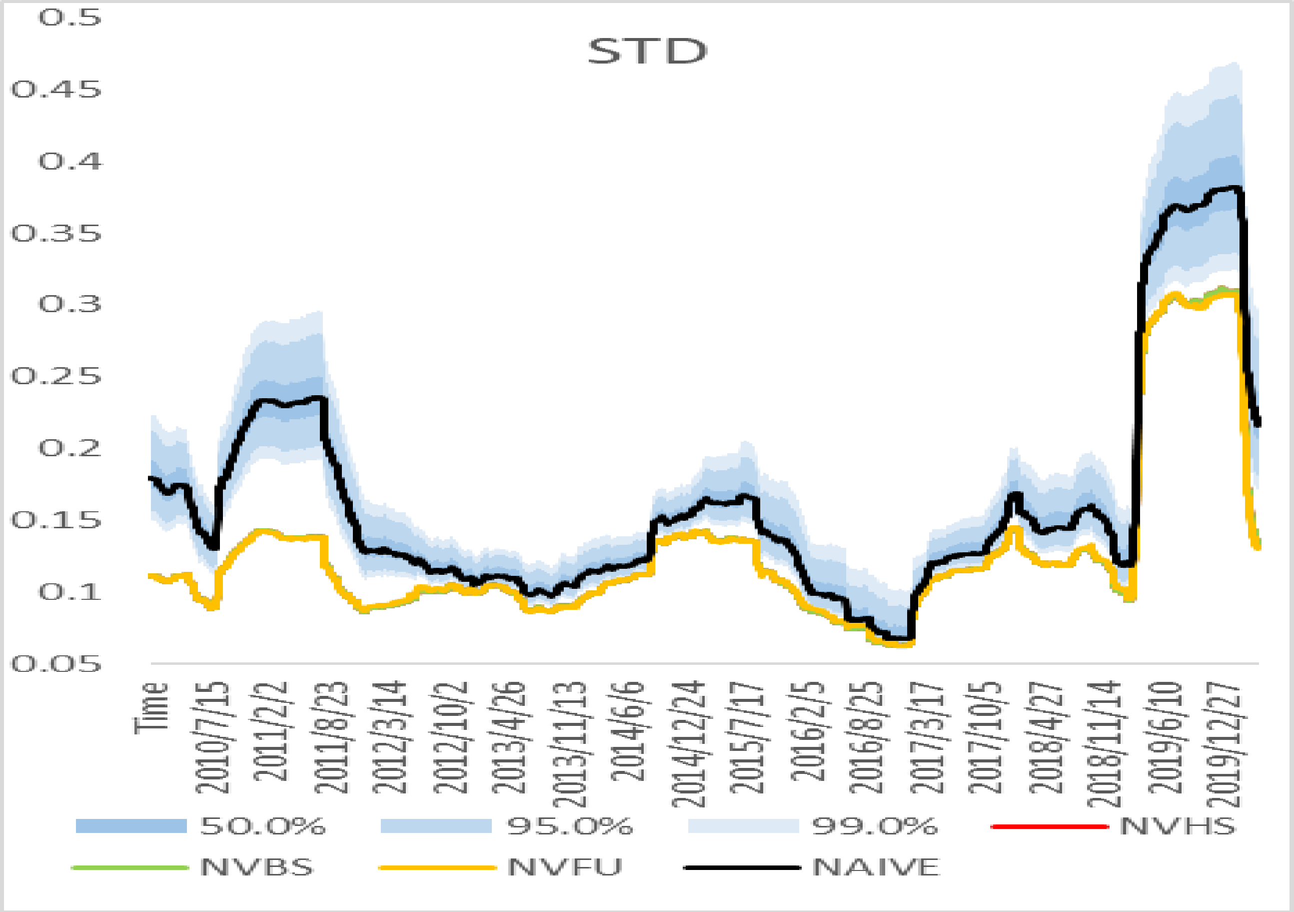
	MR	JA	TR	SR	SS08	SS2	SS25	FT05	FT2	OMG	FT10	FT001	RR5	RR50	RR75	STARR 5	CR
NVHS	43.4%	**62.7%**	**67.1%**	**57.3%**	**55.9%**	**57.2%**	**57.5%**	**55.8%**	**62.2%**	**57.5%**	45.9%	**53.4%**	**66.4%**	**56.8%**	**56.0%**	**58.4%**	**55.9%**
NVBS	43.4%	**62.7%**	**67.1%**	**57.3%**	**55.9%**	**57.2%**	**57.5%**	**55.8%**	**62.2%**	**57.5%**	45.9%	**53.4%**	**66.4%**	**56.8%**	**56.0%**	**58.4%**	**55.9%**
NVFU	41.7%	**60.8%**	**64.7%**	**55.0%**	**53.1%**	**54.6%**	**54.9%**	**52.4%**	**60.3%**	**55.5%**	45.7%	47.2%	**61.4%**	**54.5%**	**54.0%**	**55.7%**	**54.2%**
NMAD	43.2%	**62.3%**	**66.6%**	**57.0%**	**56.0%**	**57.0%**	**57.3%**	**54.6%**	**55.0%**	**51.8%**	46.6%	**55.0%**	**63.7%**	**56.6%**	**56.5%**	**58.0%**	**54.9%**
NCVAR1	46.5%	**59.6%**	**63.7%**	**56.2%**	**55.1%**	**56.5%**	**56.8%**	**51.7%**	**63.1%**	**55.2%**	**53.2%**	51.0%	**75.7%**	**55.7%**	**53.8%**	**57.6%**	**57.6%**
NCVAR5	45.8%	**64.4%**	**70.0%**	**58.8%**	**57.5%**	**58.8%**	**59.0%**	**52.7%**	**58.8%**	**54.1%**	44.1%	43.7%	**69.2%**	**58.3%**	**57.7%**	**60.2%**	**58.3%**
NCVAR10	44.6%	**64.0%**	**69.2%**	**57.7%**	**56.2%**	**57.5%**	**57.8%**	49.1%	**55.8%**	49.4%	41.3%	46.3%	**67.4%**	**57.2%**	**56.6%**	**59.1%**	**56.4%**
XRHS	**67.4%**	**69.0%**	**68.6%**	50.0%	50.3%	51.3%	**51.7%**	49.4%	**70.8%**	**60.3%**	**75.7%**	37.8%	**67.3%**	50.0%	46.5%	**51.6%**	**53.7%**
XRFU	**55.8%**	**56.4%**	**55.9%**	40.3%	40.9%	41.4%	41.5%	45.6%	**66.7%**	**57.1%**	**73.0%**	34.1%	**64.4%**	40.4%	37.3%	41.4%	42.7%
XSR	**59.9%**	**71.8%**	**74.3%**	**60.3%**	**58.4%**	**59.7%**	**60.1%**	46.5%	**51.7%**	47.1%	**53.0%**	42.8%	**61.4%**	**59.7%**	**61.1%**	**61.2%**	**58.1%**
XRR5	45.0%	44.1%	42.8%	25.3%	25.2%	25.9%	25.9%	39.5%	**57.5%**	49.7%	50.7%	42.6%	**64.8%**	25.5%	24.5%	26.4%	29.2%
XRR50	**60.2%**	**72.1%**	**74.7%**	**61.8%**	**59.9%**	**61.0%**	**61.3%**	45.8%	**51.2%**	46.4%	**53.3%**	45.4%	**62.6%**	**61.2%**	**63.2%**	**62.5%**	**59.0%**
XRR75	**64.1%**	**74.6%**	**77.4%**	**65.9%**	**63.9%**	**64.8%**	**64.9%**	50.7%	**55.3%**	**51.3%**	**51.8%**	50.0%	**57.6%**	**65.5%**	**67.0%**	**66.6%**	**61.6%**
XSTARR5	**58.5%**	**71.8%**	**74.6%**	**58.3%**	**56.2%**	**57.7%**	**58.1%**	44.1%	**51.5%**	45.9%	**53.1%**	39.2%	**58.7%**	**57.7%**	**59.6%**	**59.3%**	**56.0%**
XSS08	**61.6%**	**72.4%**	**74.7%**	**63.3%**	**61.4%**	**62.6%**	**63.0%**	48.0%	**51.5%**	46.9%	**54.1%**	48.9%	**63.2%**	**62.7%**	**64.6%**	**64.0%**	**60.4%**
XSS2	**60.8%**	**72.7%**	**75.0%**	**61.4%**	**59.2%**	**60.9%**	**61.4%**	47.0%	**51.3%**	46.8%	**53.5%**	43.2%	**62.5%**	**60.7%**	**62.3%**	**62.6%**	**59.4%**
XSS25	**60.9%**	**72.9%**	**75.0%**	**61.2%**	**59.0%**	**60.8%**	**61.3%**	46.7%	50.5%	46.5%	**53.5%**	42.3%	**61.6%**	**60.5%**	**61.5%**	**62.4%**	**59.8%**
XFT05	**75.2%**	**72.6%**	**69.8%**	52.7%	51.9%	53.3%	53.5%	**56.2%**	**62.6%**	**57.6%**	**57.3%**	**54.5%**	**71.0%**	**52.5%**	**52.0%**	**54.0%**	**57.4%**
XFT2	49.7%	**57.9%**	**59.7%**	48.4%	46.4%	48.1%	48.7%	39.5%	**52.5%**	43.4%	**59.0%**	36.2%	**59.5%**	47.6%	48.9%	49.1%	48.6%
XOMG	**62.4%**	**72.7%**	**74.5%**	**63.1%**	**61.3%**	**62.8%**	**63.2%**	49.8%	**53.9%**	49.8%	**56.6%**	48.3%	**63.2%**	**62.5%**	**63.5%**	**64.0%**	**61.3%**
XFT10	**51.9%**	**51.9%**	**52.4%**	42.2%	41.5%	42.8%	43.2%	41.3%	**61.0%**	50.7%	**63.5%**	39.9%	**60.3%**	41.9%	39.5%	43.1%	42.6%
XFT001	**56.0%**	**53.1%**	**52.1%**	32.4%	32.4%	33.7%	34.0%	**52.9%**	**62.6%**	**57.0%**	44.3%	**66.3%**	70.5%	32.2%	29.1%	34.4%	34.7%
NAIVE	49.7%	49.3%	49.3%	**54.1%**	**56.2%**	**53.7%**	**53.1%**	**54.8%**	39.3%	46.9%	36.4%	**62.6%**	39.6%	**55.2%**	**55.7%**	**52.4%**	**53.3%**

- In **bold** we highlight the mean relative rankings which indicate the strategies improved the performance measures.
- Considering the performance measures, the strategies XRR50, XRR75, XSS and XOMG perform better than the other strategies.

Empirical Studies on Portfolio Historical Performance Verification – Case Study 3

- To compare the performance of strategy portfolios with that of the random portfolios in a more intuitive way, we show the evolutions of the ranking of strategy portfolios (considering risk and performance measures) during the analyzed period

Figure: Evolution of Standard Deviation (STD) for Obtained Strategy Portfolios



- The blue areas are constructed by the different percentiles of the values of STD for all the 30,000 generated random-weights portfolios during the rolling out-of-sample periods

- The **darkest/ medium dark/ lightest blue area** represents **50%/ 95%/ 99%** of the values of STD of random portfolios around the median (i.e., the boundaries are **25th and 75th percentiles/ 2.5th and 97.5th percentiles/ 0.5th and 99.5th percentiles**

- When the value of risk measure for strategy portfolio is below the blue area, or when the value of performance measure for strategy portfolio is above the blue area, then we can consider the strategy portfolio as lowering the risk or improving the performance at **25%/ 2.5%/ 0.5%**

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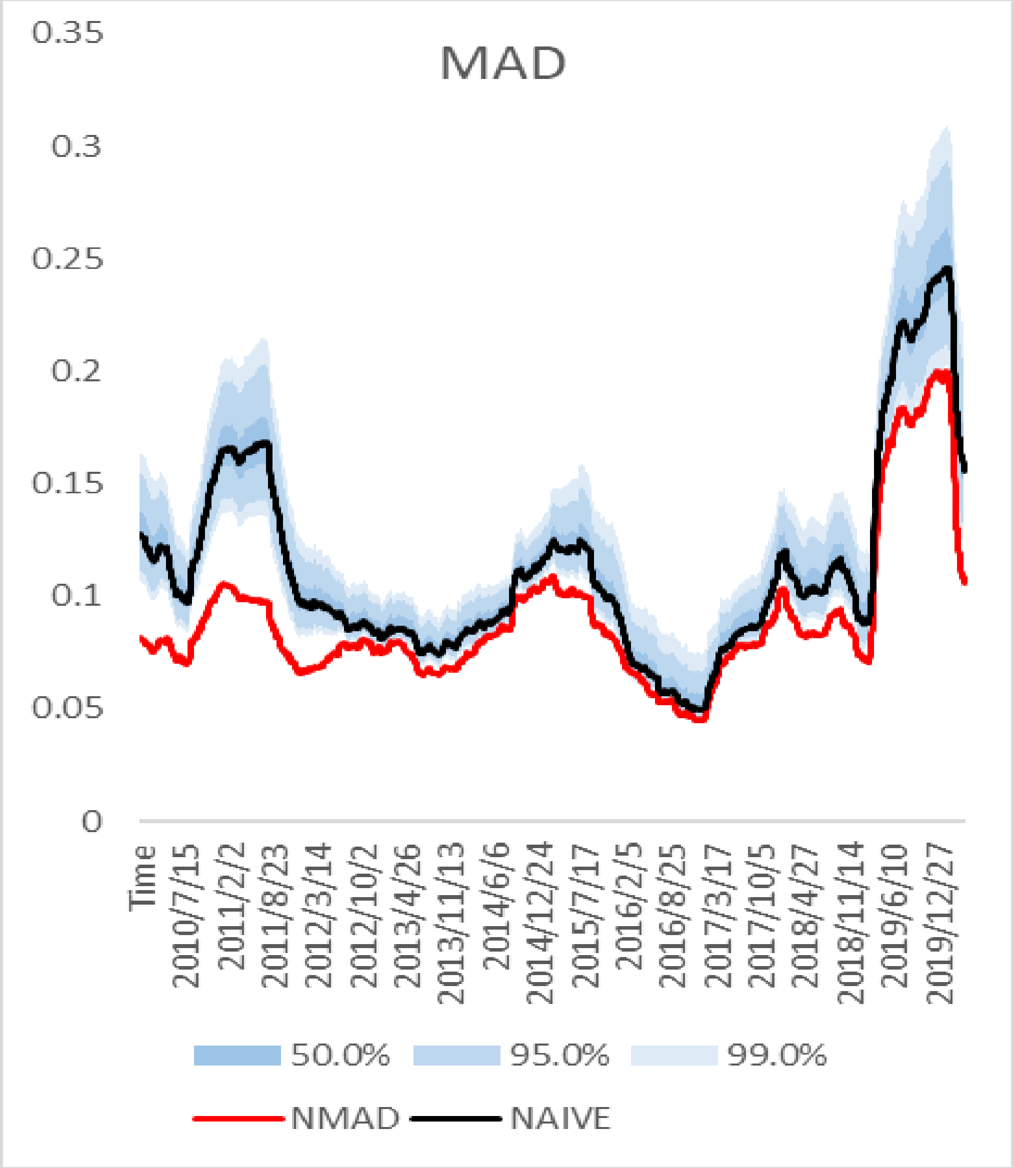
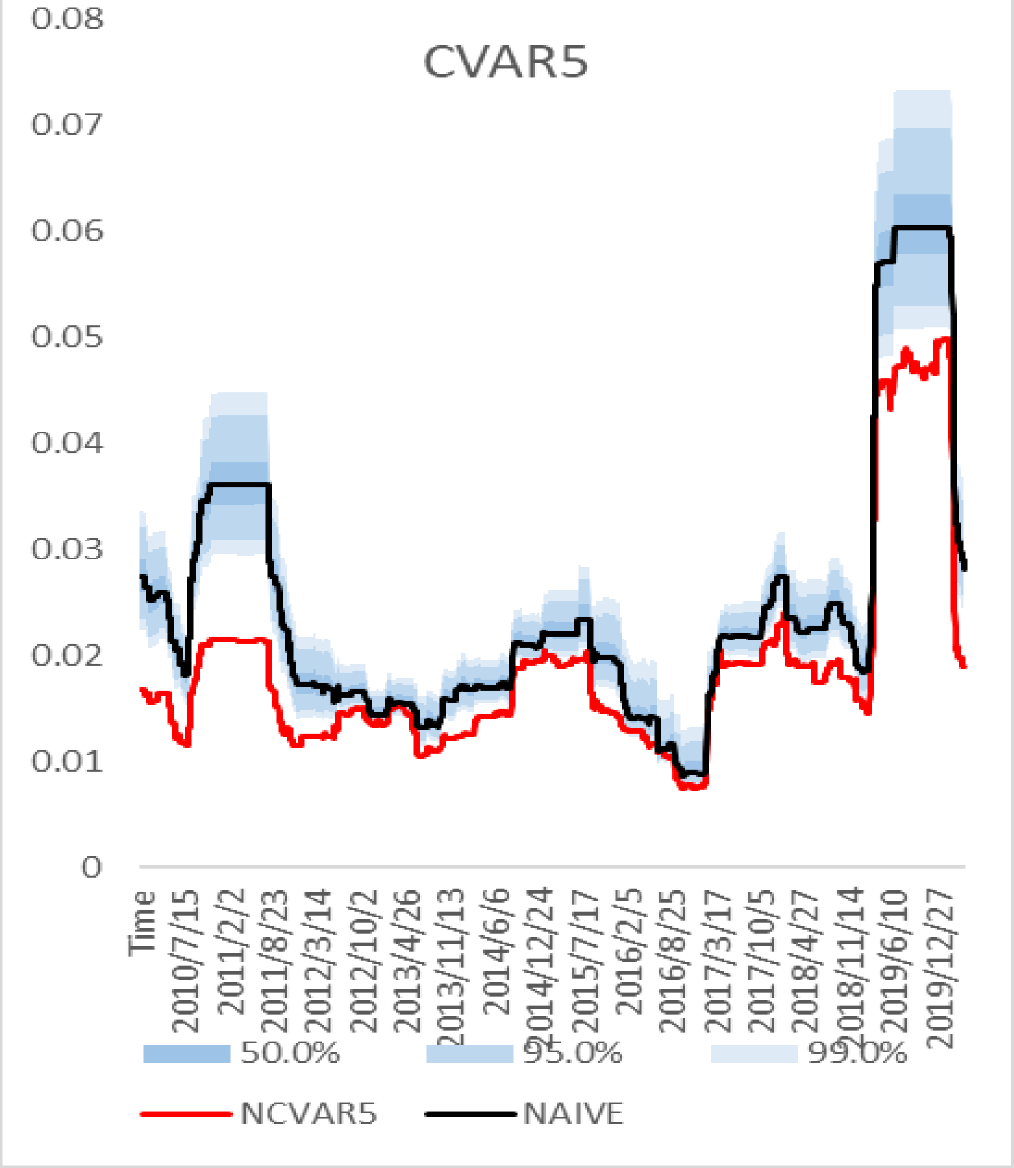
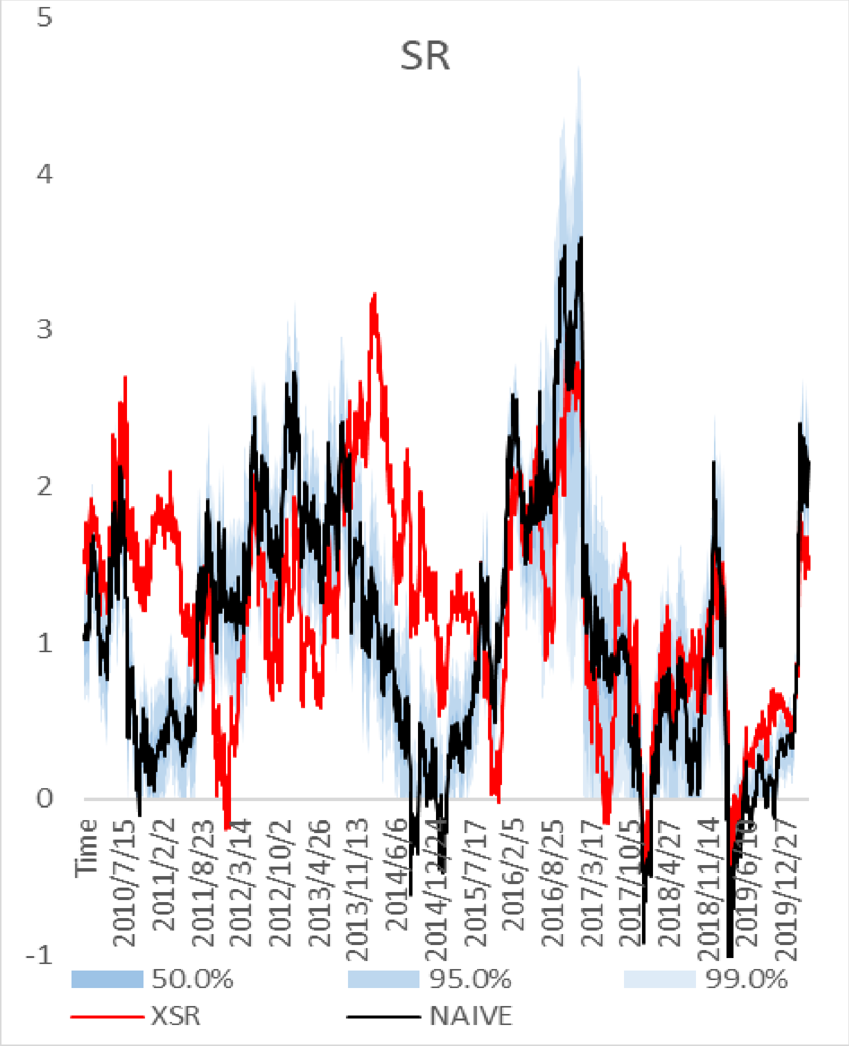
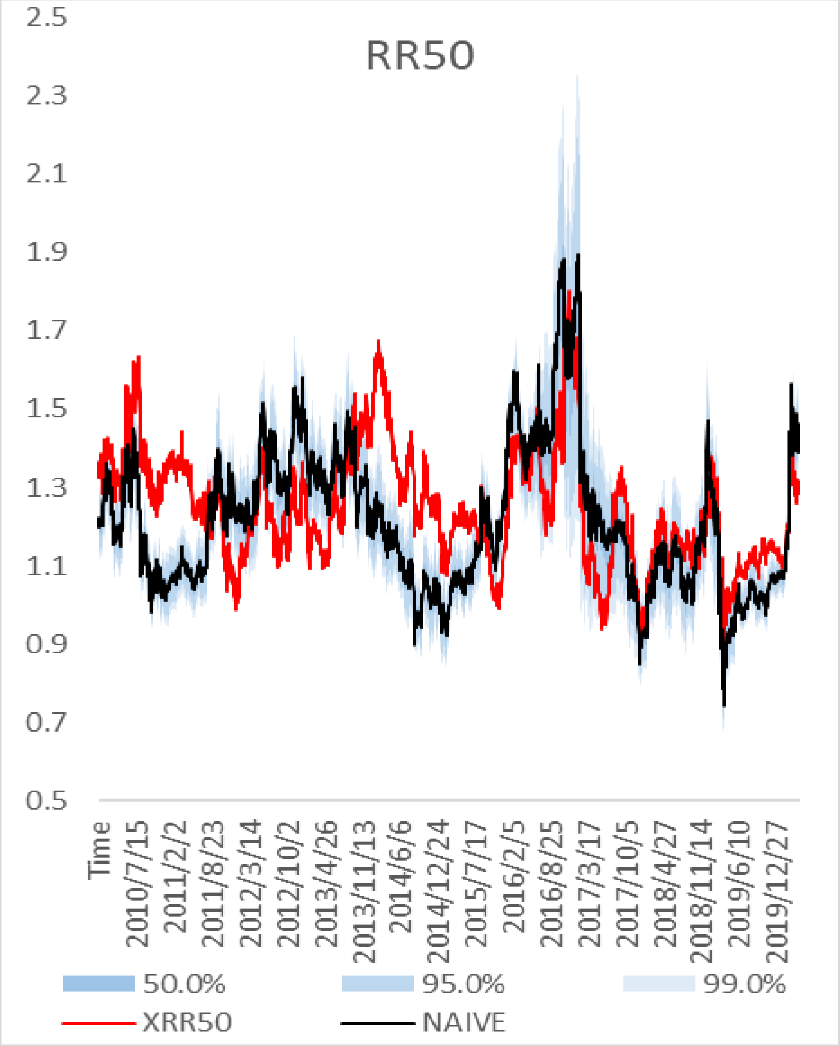


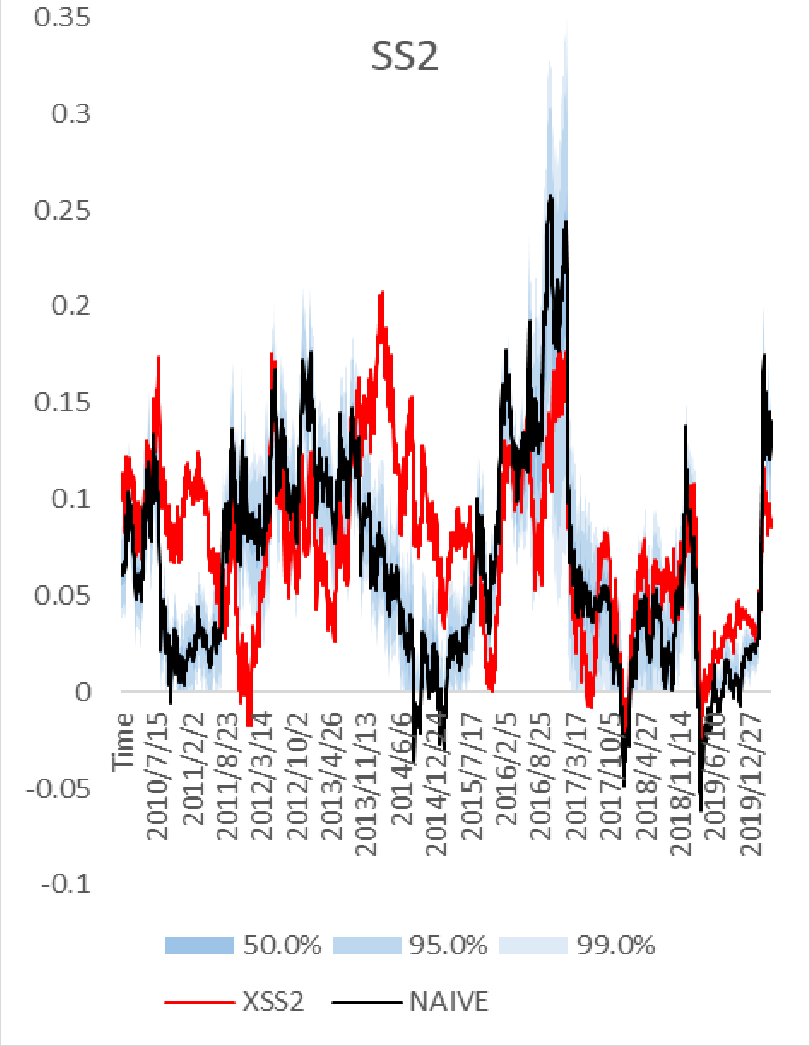


Empirical Studies on Portfolio Historical Performance Verification – Case Study 3

- Considering the performance measures, the evolutions of the ranking of strategy portfolios are blurry as the values of performance measures change from day to day







Conclusion

Based on the empirical case studies, we make three most important conclusions…

I. The strategies minimizing the classical risk measures (variance, MAD, CVaR and semivariance) in-sample performed well in the out-of-sample period in terms of the corresponding risk measures, but in terms of the performance measures, these strategy portfolios performed randomly.

II. In special, the Minimum-CVaR strategy leads to both the improved performance and the lowered risk in case of a crisis when a market changes from the uptrend to the downtrend.

III. The Maximum-Performance strategies were not verified as having a stable advantage in improving the corresponding performance measure in the out-of-sample periods, and most of them reacted similarly to the changes of periods.

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Thank you.