Partitioning of Net Ecosystem Exchange Using Dynamic Mode Decomposition and Time Delay Embedding

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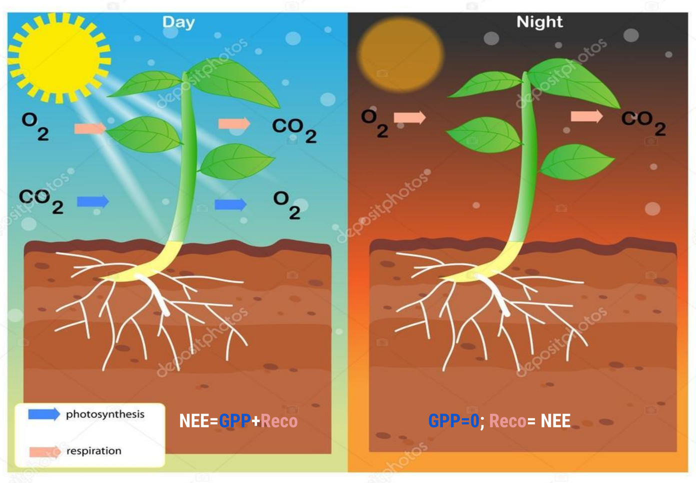
Introduction & Methodological Background

Net Ecosystem Exchange (NEE) Partitioning: Motivation & Challenges

- Better understanding of ecosystem-climate interactions and impact of climate extremes on ecosystems
- Only NEE is measured: infinite possible solutions
- NEE=GPP+Reco; GPP=0 at night
- No ground truth to compare to during day-time

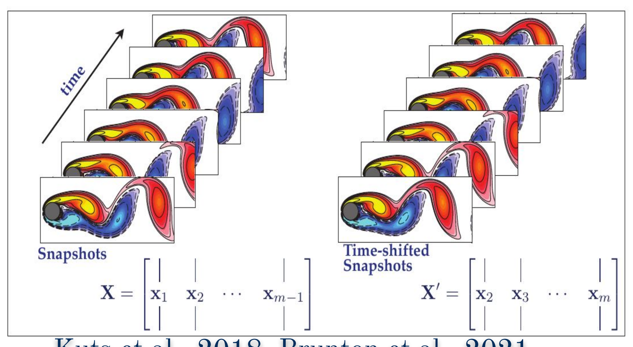


Gross primary production (GPP): CO2 used by photosynthesis
Ecosystem respiration (Reco): CO2 released by respiration

Dynamic Mode Decomposition

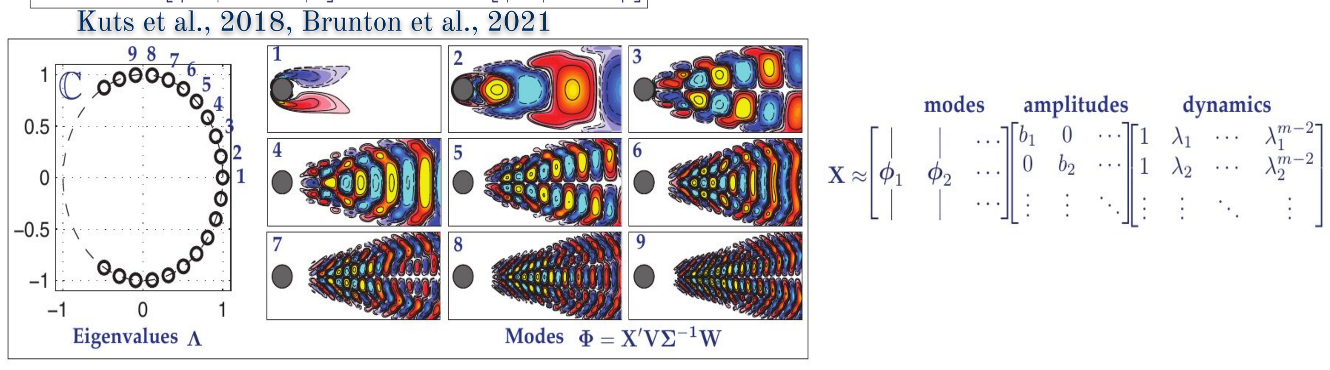
Mezic 2005,Schmid 2010

- Given a time series of data, DMD computes a set of temporal modes whereby each mode is associated with a damped (or driven) sinusoidal behavior in time.



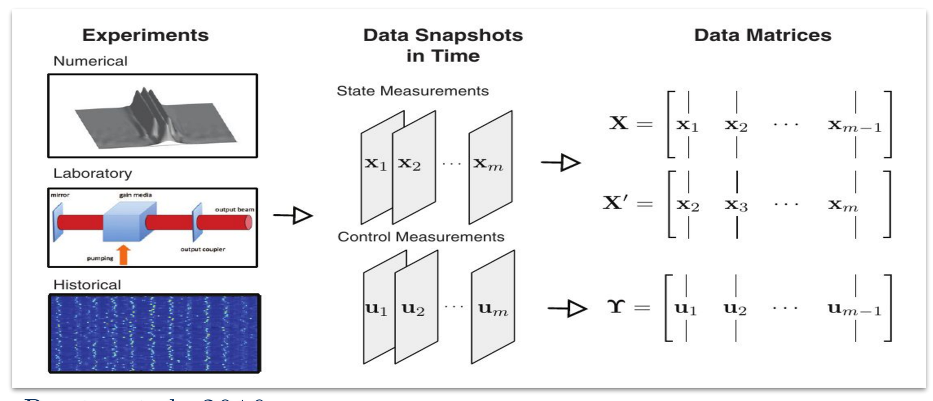
1. Approximate map $X' \approx AX$
2. Take SVD of X $X = U\Sigma V^*$
3. Reduced matrix $\tilde{A} \approx U^* X' V \Sigma^{-1}$
4. Eigendecomposition $\tilde{A}W = W\Lambda$
5. Compute modes $\Phi = X' V \Sigma^{-1} W$

Kuts et al., 2018, Brunton et al., 2021



DMD with Control (DMDc)

Proctor et al., 2016



Proctor et al., 2016

$$X' \approx AX + B\Upsilon$$

Disambiguates the natural unforced dynamics and the effect of the exogenous force input

NEE Partitioning using DMDc & Time Delay Embedding (TDE)

Data

FLUXNET2015 dataset http://fluxnet.fluxdata.org

- Sites with different vegetation type and average temperature.
- Site-years selected if gap-filled data is < 20%.

Meteorological and Eddy Covariance measurements:

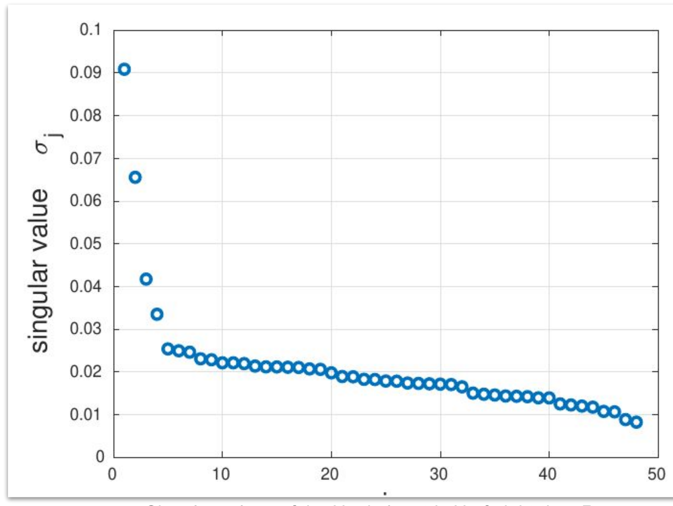
- NEE
- Fluxnet Reco-NT
- Night time index (NIGHT)
- Air temperature (Tair),
- Soil Water Content (SWC)

DMDc with TDE

$$x(k+1) = A\,x(k) + B\,u(k);$$

$x(k)$=Reco(k); $u(k)$ = temperature; soil water content

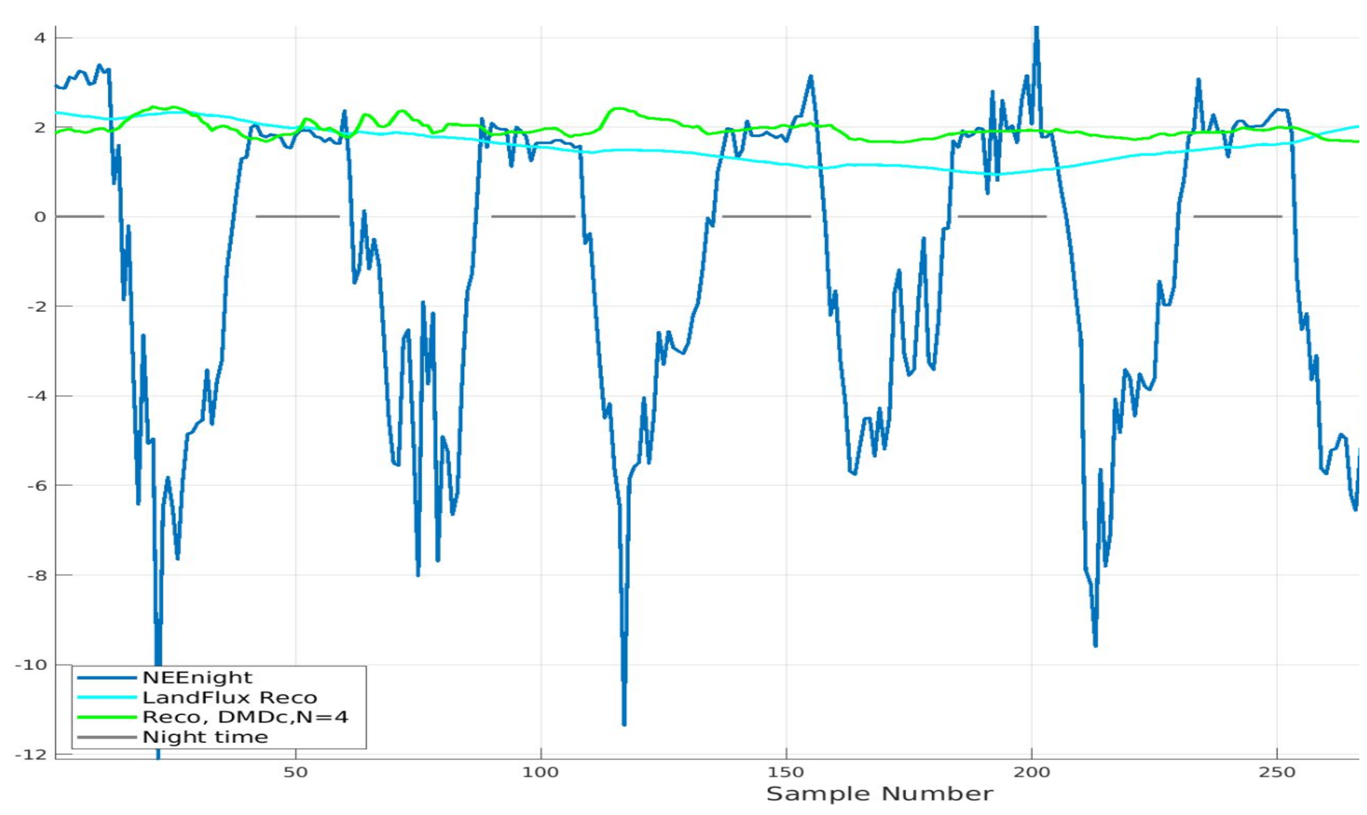
- Use of DMDc in temporal sliding window to estimate A and B from night time NEE
- Use night time model to predict day-time Reco with day-time control input
- Use TDE to compensate for unobserved drivers of Reco
- Exogenous control input (u(k)) allows to study the effect of Tair or SWC on Reco
- TDE Dimension N is selected as the rank of NEE_night Hankel matrix



Singular values of the Hankel matrix H of night-time Reco.

$$H = \begin{bmatrix} \mathrm{Reco}(t_1) & \mathrm{Reco}(t_2) & \cdots & \mathrm{Reco}(t_{M-N}) \\ \mathrm{Reco}(t_2) & \mathrm{Reco}(t_3) & \cdots & \mathrm{Reco}(t_{M-N+1}) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{Reco}(t_N) & \mathrm{Reco}(t_{N+1}) & \cdots & \mathrm{Reco}(t_M) \end{bmatrix}$$

Experimental Results



Reco 5 days forecast using DMDc with Tair as control input on site FR_Pue, France in August 2014.

- DMDc for Reco prediction has performance comparable to data driven deep learning-based methods
- SWC is better short term predictor while Tair is better long term predictor.

Table 1. Comparison of different methods' night-time Reco forecast MAPE. Training period is 3 days of night time NEE in all experiments. Forecast interval is three night-time Reco. Results are presented for DMD, DMDc, as well as DMDc with delay embedding (DMDc-TDE). Embedding dimension N=4. Vegetation type: Evergreen Broadleaf Forests (EBF), Deciduous Broadleaf Forests (DBF), Evergreen Needleleaf Forests (ENF), and Savannas (SAV).

Method (Control Input)	Fluxnet Site Name (Vegetation Type)				
	DE-Hai (DBF)	US-Ton (SAV)	IT-Ro1 (DBF)	BR-Sa3 (ENF)	NL-Loo (EBF)
DMD	0.2484	1.7187	0.3677	0.9969	1.0174
DMDc (Tair)	0.2481	**1.6682**	0.3688	0.9975	1.0153
DMDc-TDE (Tair)	0.2568	1.7240	0.3878	1.0688	**0.9580**
DMDc (SWC)	**0.2467**	1.7043	**0.3655**	**0.9963**	1.0107

Table 2. Comparison of different methods' night-time Reco forecast MAPE. Training period is 14 days of night time NEE in all experiments. Forecast interval is 14 days.

Method (Control Input)	Fluxnet Site Name (Vegetation. Type)				
	DE-Hai (DBF)	US-Ton (SAV)	IT-Ro1 (DBF)	BR-Sa3 (ENF)	NL-Loo (EBF)
DMD	0.3531	**1.9824**	0.4720	0.9705	1.0277
DMDc (Tair)	0.3559	1.9912	0.4719	0.9704	1.0351
DMDc-TDE (Tair)	**0.3471**	2.0204	**0.4641**	1.0218	**1.0048**
DMDc (SWC)	0.3515	1.9915	0.4742	**0.9701**	1.0265

Acknowledgment: This study is supported by DFG Research Grant SH 1682/1-1, and the Carl Zeiss Foundation within the scope of the program line "Breakthroughs: Exploring Intelligent Systems" for "Digitization — explore the basics, use applications".

Carl Zeiss Stiftung
MichaelStifelCenterJena for Data-Driven and Simulation Science
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