RECOMMENDATIONS OF STOCKBROKERS VS. FUZZY PORTFOLIO APPROACH IN CONSTRUCTION SECTOR

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ITISE 2022 (8th International conference on Time Series and Forecasting), 27th-30th June, Gran Canaria, Spain

AGENDA

1. Introduction
2. Trapezoidal Oriented Fuzzy numbers
3. Portfolio approach
4. Case study
5. Conclusions

FUZZY NUMBERS - DEFINITIONS

- Fuzzy numbers allow us to formally describe and process non-precision quantities and information.
- Imprecise number – family of values in which each considered value belongs to it in varying degree.
- Fuzzy number – fuzzy set on the real line $\mathbb{R}$.

TRAPEZOIDAL ORIENTED FUZZY NUMBERS

DEFINITION (PIASECKI, 2018)

For any monotonic sequence $\{a, b, c, d\} \subset \mathbb{R}$ **trapezoidal oriented fuzzy number** (TrOFN) $\overleftrightarrow{Tr}(a, b, c, d)$ is defined as a pair of fuzzy numbers determined by their membership function $\mu_{\overleftrightarrow{Tr}}(\cdot\,|a, b, c, d) \in [0,1]^{\mathbb{R}}$ given by the identity

$$\mu_T(x) = \mu_{Tr}(x|a, b, c, d) = \begin{cases} 0, & x \notin [a, d] \equiv [d, a], \\ \frac{x-a}{b-a}, & x \in [a, b[\equiv]a, b], \\ 1, & x \in [b, c] \equiv [c, b], \\ \frac{x-d}{c-d}, & x \in]c, d] \equiv [c, d[. \end{cases}$$

and orientation from $a \in \mathbb{R}$ to $b \in \mathbb{R}$.

EXTENDED BASIC ARITHMETIC OPERATORS TO THE CASE OF $\mathbb{K}_{Tr}$

- $\mathbb{K}_{Tr}$ - the space of all TrOFNs
- $\mathbb{K}_{Tr}^{+}$ - the space of all positively oriented TrOFNs
- $\mathbb{K}_{Tr}^{-}$ - the space of all negatively oriented TrOFNs.

For any pair $\left(\overleftrightarrow{Tr}(a,b,c,d), \overleftrightarrow{Tr}(p-a,q-b,r-c,s-d)\right) \in \mathbb{K}_{Tr}^{2}$ and $\beta \in \mathbb{R}$

$$\overleftrightarrow{Tr}(a,b,c,d) \boxplus \overleftrightarrow{Tr}(p-a,q-b,r-c,s-d) \; =$$

$$\begin{cases} \overleftrightarrow{Tr}(\min\{p,q\},q,r,\max\{r,s\}), & (q<r) \vee (q=r \wedge p \leq s), \\ \overleftrightarrow{Tr}(\max\{p,q\},q,r,\min\{r,s\}), & (q>r) \vee (q=r \wedge p>s), \end{cases}$$

$$\beta \boxdot \overleftrightarrow{Tr}(a,b,c,d) = \overleftrightarrow{Tr}(\beta \cdot a, \beta \cdot b, \beta \cdot c, \beta \cdot d).$$

IMPRECISION

Imprecision - a superposition of ambiguity and indistinctness of information.

- **Ambiguity** can be interpreted as a lack of a clear recommendation between one alternative among various others.
- **Indistinctness** is understood as a lack of explicit distinction between recommended and not recommended alternatives.

Any OFN is a particular kind of imprecision information.

An increase in information imprecision reduces suitability of this information.

FUTURE VALUE, PRESENT VALUE, RATE OF RETURN

Let us assume that the time horizon $t > 0$ of an investment is fixed.

Then, the asset considered here is determined by two values:

- anticipated future value (FV) V_t,
- assessed present value (PV) V_0.

The basic characteristic of benefits from owning this asset is a simple return rate r_t given by the identity:

$$r_t = \frac{V_t - V_0}{V_0} = \frac{V_t}{V_0} - 1.$$

EXPECTED DISCOUNT FACTOR

FV is a random variable $\tilde{V}_t: \Omega \longrightarrow \mathbb{R}^+$, where Ω, is - a set of elementary states ω, of the financial market.

In a classical approach to a return rate estimation, PV is identified with the observed quoted price $\check{P}$.

Thus, the return rate is a random variable determined by identity:

$$r_t(\omega) = \frac{\tilde{V}_t(\omega) - \check{P}}{\check{P}}.$$

EXPECTED DISCOUNT FACTOR

We assume that the expected value, $\bar{r}$, of this distribution exists.

Then the expected discount factor (EDF) $\bar{v}$ exists and is determined by the dependency

$$\bar{v} = (1 + \bar{r})^{-1}.$$

Then

$$r_t = r_t(V_0, \omega) = \frac{\check{P} \cdot (1 + r_t(\omega))}{V_0} - 1.$$

We determine the imprecise EDF $\mathcal{V}: \mathbb{R}^+ \to \mathbb{R}^+$ as a unary operator transforming PV as follows

$$\mathcal{V}(V_0) = \left(\frac{\check{P} \cdot (1 + \bar{r})}{V_0}\right)^{-1} = \frac{\bar{v}}{\check{P}} \cdot V_0.$$

IMPRECISE PRESENT VALUE

If PV is imprecisely estimated, then it may be evaluated by TrOFN

$$\overleftrightarrow{PV} = \overleftrightarrow{Tr}(V_s, V_f, V_l, V_e),$$

where the monotonic sequence $(V_s, V_f, \check{P}, V_l, V_e)$ is determined in a following way

- $[V_s, V_e] \subset \mathbb{R}^+$ is an interval of all possible values of PV,
- $[V_f, V_l] \subset [V_s, V_e]$ is an interval of all prices which do not noticeably differ from a quoted price $\check{P}$.

If we predict a rise in price, then PV is described by a positively oriented TrOFN.

If we predict a fall in price, then PV is described by a negatively oriented TrOFN.

IMPRECISE EDF

Using Kosinski's approach, we define the imprecise EDF by an extension $\overleftrightarrow{\mathcal{V}}: \mathbb{K}_{Tr} \to \mathbb{K}$ of a unary operator and the imprecise EDF $\overleftrightarrow{\mathcal{V}}(\overleftrightarrow{PV})$ is given by TrOFN

$$\overleftrightarrow{\mathcal{V}}(\overleftrightarrow{PV}) = \overleftrightarrow{Tr}\left(\frac{V_s \cdot \bar{v}}{\check{P}}, \frac{V_f \cdot \bar{v}}{\check{P}}, \frac{V_l \cdot \bar{v}}{\check{P}}, \frac{V_e \cdot \bar{v}}{\check{P}}\right) = \overleftrightarrow{\mathcal{V}}\left(\overleftrightarrow{Tr}(V_s, V_f, V_l, V_e)\right).$$

Analogous extension $\overleftrightarrow{\mathcal{R}}(\overleftrightarrow{PV})$ is such OFN which is not TrOFN.

In considered case it causes that for portfolio analysis, imprecise EDF is more useful than imprecise expected return rate.

MULTI-ASSET PORTFOLIO

Let us consider the case of a multi-asset portfolio π^*, built of securities Y_i .

We describe this portfolio as the set $\pi^* = \{Y_i : i = 1,2, \dots, n\}$.

Any security Y_i is characterized by its price $\check{P}_i \in \mathbb{R}^+$, by its imprecise PV evaluated by TrOFN

$$\overleftrightarrow{PV_i} = \overleftrightarrow{Tr}\left(V_s^{(i)}, V_f^{(i)}, V_l^{(i)}, V_e^{(i)}\right)$$

and by its EDF $\bar{v}_i$ determined by

$$\bar{v} = (1 + \bar{r})^{-1}.$$

We evaluate any security Y_i by its imprecise EDF

$$\overleftrightarrow{\mathcal{V}_i} = \overleftrightarrow{Tr}\left(D_s^{(i)}, D_f^{(i)}, D_l^{(i)}, D_e^{(i)}\right) = \overleftrightarrow{Tr}\left(V_s^{(i)} \cdot \frac{\bar{v}_i}{\check{P}_i}, V_f^{(i)} \cdot \frac{\bar{v}_i}{\check{P}_i}, V_l^{(i)} \cdot \frac{\bar{v}_i}{\check{P}_i}, V_e^{(i)} \cdot \frac{\bar{v}_i}{\check{P}_i}\right).$$

MULTI-ASSET PORTFOLIO

We distinguish the portfolio of rising securities $\pi^+ = \{Y_i \in \pi^* : \overleftrightarrow{PV}_i \in \mathbb{K}_{Tr}^+\}$

and the portfolio of falling securities $\pi^- = \pi^* \backslash \pi^+$.

Then, we calculate the PV of portfolio π^+

$$\overleftrightarrow{PV}^+ = \overleftrightarrow{Tr}\left(V_s^{(+)}, V_f^{(+)}, V_l^{(+)}, V_e^{(+)}\right) = \overleftrightarrow{Tr}\left(\sum_{Y_i \in \pi^+} V_s^{(i)}, \sum_{Y_i \in \pi^+} V_f^{(i)}, \sum_{Y_i \in \pi^+} V_l^{(i)}, \sum_{Y_i \in \pi^+} V_e^{(i)}\right)$$

and the PV of portfolio π^-

$$\overleftrightarrow{PV}^- = \overleftrightarrow{Tr}\left(V_s^{(-)}, V_f^{(-)}, V_l^{(-)}, V_e^{(-)}\right) = \overleftrightarrow{Tr}\left(\sum_{Y_i \in \pi^-} V_s^{(i)}, \sum_{Y_i \in \pi^-} V_f^{(i)}, \sum_{Y_i \in \pi^-} V_l^{(i)}, \sum_{Y_i \in \pi^-} V_e^{(i)}\right).$$

Finally, we calculate the PV of portfolio π^*, denoted by the symbol $\overleftrightarrow{PV}^*$. We get

$$\overleftrightarrow{PV}^* = \overleftrightarrow{PV}^+ \boxplus \overleftrightarrow{PV}^- = \overleftrightarrow{Tr}\left(V_s^{(+)}, V_f^{(+)}, V_l^{(+)}, V_e^{(+)}\right) \boxplus \overleftrightarrow{Tr}\left(V_s^{(-)}, V_f^{(-)}, V_l^{(-)}, V_e^{(-)}\right).$$

EDF OF CONSIDERED PORTFOLIO

Now we can start calculating EDFs of considered portfolios. The values of portfolios π^+, π^-, π^* are respectively calculated in following way

$$M^+ = \sum_{Y_i \in \pi^+} \check{P}_i, \qquad M^- = \sum_{Y_i \in \pi^-} \check{P}_i, \qquad M^* = M^+ + M^-.$$

The share p_i^+ of the asset $Y_i \in \pi^+$ in the portfolio π^+ and the share p_i^- of the asset $Y_i \in \pi^-$ in the portfolio π^- are given by formulas

$$p_i^+ = \frac{\check{P}_i}{M^+}, \quad p_i^- = \frac{\check{P}_i}{M^-}.$$

The share q^+ of portfolio π^+ in the portfolio π^* and the share q^- of portfolio π^- in the portfolio π^* are given by formulas

$$q^+ = \frac{M^+}{M^*}, \quad q^- = \frac{M^-}{M^*}.$$

EDF OF CONSIDERED PORTFOLIO

The EDF $\bar{v}^+$ of portfolio π^+, the EDF $\bar{v}^-$ of portfolio π^- and the EDF $\bar{v}^*$ of portfolio π^* are calculated as follows

$$\bar{v}^+ = \left(\sum_{Y_i \in \pi^+} \frac{p_i^+}{\bar{v}_i}\right)^{-1}, \qquad \bar{v}^- = \left(\sum_{Y_i \in \pi^-} \frac{p_i^-}{\bar{v}_i}\right)^{-1}, \qquad \bar{v}^* = \left(\frac{q^+}{\bar{v}^+} + \frac{q^-}{\bar{v}^-}\right)^{-1}.$$

EDF OF CONSIDERED PORTFOLIO

- The imprecise EDF $\overrightarrow{\mathcal{V}}^{+}$ of portfolio π^{+} is given by the formula

$$\overrightarrow{\mathcal{V}}^{+} = \overrightarrow{Tr}\left(\sum_{Y_i \in \pi^{+}} \frac{\bar{v}^{+} \cdot p_i^{(+)}}{\bar{v}_i} \cdot D_s^{(i)}, \sum_{Y_i \in \pi^{+}} \frac{\bar{v}^{+} \cdot p_i^{(+)}}{\bar{v}_i} \cdot D_f^{(i)}, \sum_{Y_i \in \pi^{+}} \frac{\bar{v}^{+} \cdot p_i^{(+)}}{\bar{v}_i} \cdot D_l^{(i)}, \sum_{Y_i \in \pi^{+}} \frac{\bar{v}^{+} \cdot p_i^{(+)}}{\bar{v}_i} \cdot D_e^{(i)}\right).$$

- The imprecise EDF $\overrightarrow{\mathcal{V}}^{-}$ of portfolio π^{-} is given by the formula

$$\overrightarrow{\mathcal{V}}^{-} = \overrightarrow{Tr}\left(\sum_{Y_i \in \pi^{-}} \frac{\bar{v}^{-} \cdot p_i^{(-)}}{\bar{v}_i} \cdot D_s^{(i)}, \sum_{Y_i \in \pi^{-}} \frac{\bar{v}^{-} \cdot p_i^{(-)}}{\bar{v}_i} \cdot D_f^{(i)}, \sum_{Y_i \in \pi^{-}} \frac{\bar{v}^{-} \cdot p_i^{(-)}}{\bar{v}_i} \cdot D_l^{(i)}, \sum_{Y_i \in \pi^{-}} \frac{\bar{v}^{-} \cdot p_i^{(-)}}{\bar{v}_i} \cdot D_e^{(i)}\right).$$

- The imprecise EDF $\overrightarrow{\mathcal{V}}^{*}$ of portfolio π^{*} is given by the formula

$$\overrightarrow{\mathcal{V}}^{*} = \overrightarrow{Tr}\left(D_s^{(*)}, D_f^{(*)}, D_l^{(*)}, D_e^{(*)}\right) = \left(\frac{\bar{v}^{*} \cdot q^{+}}{\bar{v}^{+}} \boxdot \overrightarrow{\mathcal{V}}^{+}\right) \boxplus \left(\frac{\bar{v}^{*} \cdot q^{-}}{\bar{v}^{-}} \boxdot \overrightarrow{\mathcal{V}}^{-}\right).$$

CASE STUDY

We observe the portfolio π^* composed of 5 companies in construction sector shares of which are included in WIG (main index of Warsaw Stock Exchange, WSE).
The portfolio π^* contains the following securities:

- Y_{COM} - 1600 stocks of COMPREMUM (COM),
- Y_{MST} - 950 stocks of MOSTOSTALZAB (MST),
- Y_{PEK} - 550 stocks of PEKABEX (PEK),
- Y_{SEL} - 1005 stocks of SELENAFM (SEL),
- Y_{SNK} - 120 stocks of ŚNIEŻKA (SNK).

Based on a session closing on the WSE on September 8, 2021, for each observed share we assess its PV equal to TrOFN $\overleftrightarrow{PV_i}$ describing its Japanese candles. A quoted price $\check{P}_S$ of each portfolio element becomes an initial price on September 9, 2021.

CASE STUDY

Assets	Present Value $\overleftrightarrow{PV}_s$	Quoted Price $\breve{P}_s$	OEDF $\overleftrightarrow{V}_s$ – energy measure	OEDF $\overleftrightarrow{V}_s$ - entropy measure
Y_{COM}	$\overleftrightarrow{Tr}(4.86;\ 4.92;\ 5.04;\ 5.06)$	5.02	0.0305	0.0038
Y_{MST}	$\overleftrightarrow{Tr}(1.85;\ 1.89;\ 2.03; 2.14)$	2.04	0.1007	0.0176
Y_{PEK}	$\overleftrightarrow{Tr}(23.50;\ 23.10;\ 22.90;\ 22.90)$	22.70	0.0168	0.0042
Y_{SEL}	$\overleftrightarrow{Tr}(29.00;\ 29.00; 27.90;\ 27.20)$	28.00	0.0495	0.0060
Y_{SNK}	$\overleftrightarrow{Tr}(84.20;\ 84.20;\ 82.00;\ 81.80)$	82.00	0.0268	0.0006

CASE STUDY

Assets	Present Value $\overleftrightarrow{PV}_s$	Quoted Price $\breve{P}_s$	OEDF $\overleftrightarrow{V}_s$ – energy measure	OEDF $\overleftrightarrow{V}_s$ - entropy measure
Y_{COM}	$\overleftrightarrow{Tr}(4.86;\ 4.92;\ 5.04;\ 5.06)$	5.02	0.0305	0.0038
Y_{MST}	$\overleftrightarrow{Tr}(1.85;\ 1.89;\ 2.03;\ 2.14)$	2.04	0.1007	0.0176
Y_{PEK}	$\overleftrightarrow{Tr}(23.50;\ 23.10;\ 22.90;\ 22.90)$	22.70	0.0168	0.0042
Y_{SEL}	$\overleftrightarrow{Tr}(29.00;\ 29.00;\ 27.90;\ 27.20)$	28.00	0.0495	0.0060
Y_{SNK}	$\overleftrightarrow{Tr}(84.20;\ 84.20;\ 82.00;\ 81.80)$	82.00	0.0268	0.0006

CASE STUDY

We calculate PVs of portfolios π^+, π^-, π^*:

$$\overleftrightarrow{PV}^+ = \overleftrightarrow{Tr}(9533.50, 9667.50, 9992.50, 10129.00),$$

$$\overleftrightarrow{PV}^- = \overleftrightarrow{Tr}(52174.00, 51954.00, 50474.50, 49747.00),$$

$$\overleftrightarrow{PV}^* = \overleftrightarrow{Tr}(61707.50, 61621.50, 60467.00, 59876.00).$$

The PVs of portfolio is estimated by a ***negatively*** oriented TrOFN, therefore we expect the **decrease** in value of the portfolio.

EDFs of portfolios ${\pi_1}^+$, ${\pi_1}^-$, ${\pi_1}^*$ are equal $\bar{v}^+ = 0.9854$, $\bar{v}^- = 0.9675$, $\bar{v}^* = 0.9704$.

CASE STUDY

Finally, we calculate imprecise EDFs of portfolios ${\pi_1}^+$, ${\pi_1}^-$, ${\pi_1}^*$

$$\vec{\mathcal{V}}^+ = \overleftrightarrow{Tr}(0.9134,0.9263,0.9580,0.9716),$$

$$\vec{\mathcal{V}}^- = \overleftrightarrow{Tr}(0.9882,0.9839,0.9562,0.9428),$$

$$\vec{\mathcal{V}}^* = \overleftrightarrow{Tr}(0.9760,0.9745,0.9565,0.9475).$$

The energy measure of EDF $\vec{\mathcal{V}}^*$ of portfolio π^* equals $d(\vec{\mathcal{V}}^*) = 0.0233$.

The entropy measure of EDF $\vec{\mathcal{V}}^*$ of portfolio π^* is $e(\vec{\mathcal{V}}^*) = 0.0026$.

Linear portfolio analysis is not possible for considered portfolio ${\pi_1}^*$.

CASE STUDY

Present values of Y_{COM} and Y_{MST} are estimated by a ***positively*** oriented TrOFN, hence, we expect an **increase** of their value in the future. That suggests that in the future we will have an opportunity to sell the shares with a profit.

Present values of Y_{PEK}, Y_{SEL} and Y_{SNK} are esitmated by a ***negatively*** oriented TrOFNs therefore we expect a **decrease** in their prices. That suggests a future opportunity to buy their shares at a lower price.

Basing on those premises we recommend to the investor to buy additional shares of Y_{COM} and Y_{MST} and to sell some shares of Y_{PEK}, Y_{SEL} and Y_{SNK}.

CASE STUDY

The investor decided to ask a broker for the recommendation (those presented below are official recommendations published by brokerage houses).

The recommendations for chosen companies in construction sector were as follows:

Company	Recommendation	Date of recommendation
ŚNIEŻKA	SELL	2021-09-08
MOSTALZAB	BUY	2021-09-07
PEKABEX	HOLD	2021-09-07
SELENAFM	BUY	2021-09-07
COMPREMUM	HOLD	2021-09-06

Recommendations of the broker differ insignificantly from those resulting from TrOFNs, however we will consider **two possible strategies**.

In case when the recommendations are identical in both strategies, the investor makes identical decisions concerning given shares.

STRATEGY NO. 1

Strategy No. 1 – broker's recommendation is a dominant one

Investors decides to:

1. **Sell** 20 shares of SNK
2. **Buy** 50 shares of MST
3. **Buy** 5 shares of SEL

The number of shares of remaining companies stays unchanged.

STRATEGY NO. 1

We have a portfolio π^* containing:

- Y_{COM} - 1600 stocks of COMPREMUM (COM),
- Y_{MST} - **1000** stocks of MOSTOSTALZAB (MST),
- Y_{PEK} - 550 stocks of PEKABEX (PEK),
- Y_{SEL} - **1010** stocks of SELENAFM (SEL),
- Y_{SNK} - **100** stocks of ŚNIEŻKA (SNK).

Based on a session closing on the WSE on September 10, 2021, for each observed share we assess its PV equal to TrOFN $\overleftrightarrow{PV_i}$ describing its Japanese candles.

A quoted price $\check{P}_S$ of each portfolio element becomes an initial price on September 11, 2021.

STRATEGY NO. 1

We calculate PVs of portfolios π^+, π^-, π^*:

$$\overleftrightarrow{PV}^+ = \overleftrightarrow{Tr}(28281.00, 28391.00, 28996.00, 29280.00),$$

$$\overleftrightarrow{PV}^- = \overleftrightarrow{Tr}(30651.00, 30611.00, 30107.00, 29582.00),$$

$$\overleftrightarrow{PV}^* = \overleftrightarrow{Tr}(58932.00, 59002.00, 59103.00, 59103.00).$$

The PVs of portfolio is estimated by a positively oriented TrOFN therefore we expect the **increase** in portfolio's value.

EDFs of portfolios ${\pi_1}^+$, ${\pi_1}^-$, ${\pi_1}^*$ are equal $\bar{v}^+ = 0.9492$, $\bar{v}^- = 0.9947$, $\bar{v}^* = 0.9718$.

STRATEGY NO. 1

Finally, we calculate imprecise EDFs of portfolios ${\pi_1}^+$, ${\pi_1}^-$, ${\pi_1}^*$

$$\vec{\mathcal{V}}^+ = \overleftrightarrow{Tr}(0.9314, 0.9350, 0.9549, 0.9642),$$

$$\vec{\mathcal{V}}^- = \overleftrightarrow{Tr}(0.9784, 0.9771, 0.9608, 0.9441),$$

$$\vec{\mathcal{V}}^* = \overleftrightarrow{Tr}(0.9547, 0.9559, 0.9579, 0.9579)..$$

The energy measure of EDF $\vec{\mathcal{V}}^*$ of portfolio π^* equals $d(\vec{\mathcal{V}}^*) = 0.02541.$

The entropy measure of EDF $\vec{\mathcal{V}}^*$ of portfolio π^* is $e(\vec{\mathcal{V}}^*) = 0.000292.$

Linear portfolio analysis is not possible for considered portfolio ${\pi_1}^*$

ENERGY AND ENTROPY MEASURES OF INDIVIDUAL SHARES

Assets	Present Value $\overleftrightarrow{PV}_s$	Quoted Price $\breve{P}_s$	OEDF $\overleftrightarrow{V}_s$ – energy measure	OEDF $\overleftrightarrow{V}_s$ - entropy measure
Y_{COM}	$\overleftrightarrow{Tr}(5.06;\ 5.06;\ 5.16;\ 5.20)$	5.18	0.0221	0.0018
Y_{MST}	$\overleftrightarrow{Tr}(2.27;\ 2.23;\ 2.13;\ 2.11)$	2.08	0.0597	0.0069
Y_{PEK}	$\overleftrightarrow{Tr}(21.90;\ 22.10;\ 22.80;\ 23.20)$	22.80	0.0419	0.0063
Y_{SEL}	$\overleftrightarrow{Tr}(28.10;\ 28.10;\ 27.70;\ 27.20)$	27.60	0.0225	0.0043
Y_{SNK}	$\overleftrightarrow{Tr}(81.40;\ 81.40;\ 82.00;\ 82.00)$	82.00	0.0070	0.0000

STRATEGY NO. 2

Strategy No. 2 – The investor decided to follow the recommendations resulting from the TrOFN portfolo calculations.

OFN recommendations are dominant.

Investors decides to:

1. **Sell** 20 shares of SNK
2. **Sell** 50 shares of PEK
3. **Sell** 5 shares of SEL
4. **Buy** 50 shares of MST
5. **Buy** 400 shares of COM

STRATEGY NO. 2

We have a portfolio π^* containing:

- Y_{COM} - **2000** stocks of COMPREMUM (COM),
- Y_{MST} - **1000** stocks of MOSTOSTALZAB (MST),
- Y_{PEK} - **500** stocks of PEKABEX (PEK),
- Y_{SEL} - **1000** stocks of SELENAFM (SEL),
- Y_{SNK} - **100** stocks of ŚNIEŻKA (SNK).

Based on a session closing on the WSE on September 10, 2021, for each observed share we assess its PV equal to TrOFN $\overleftrightarrow{PV_i}$ describing its Japanese candles.

A quoted price $\check{P}_S$ of each portfolio element becomes an initial price on September 11, 2021.

STRATEGY NO. 2

We calculate PVs of portfolios π^+, π^-, π^*:

$$\overleftrightarrow{PV}^+ = \overleftrightarrow{Tr}(29210.00, 29310.00, 29920.00, 30200.00),$$

$$\overleftrightarrow{PV}^- = \overleftrightarrow{Tr}(30370.00, 30330.00, 29830.00, 293102.00),$$

$$\overleftrightarrow{PV}^* = \overleftrightarrow{Tr}(59580.00, 59640.00, 59750.00, 59750.00).$$

The PVs of portfolio is estimated by a positively oriented TrOFN, therefore we expect further **increase** in portfolio's value.

EDFs of portfolios ${\pi_1}^+$, ${\pi_1}^-$, ${\pi_1}^*$ are equal $\bar{v}^+ = 0.9523$, $\bar{v}^- = 0.9945$, $\bar{v}^* = 0.9733$.

STRATEGY NO. 2

Finally, we calculate imprecise EDFs of portfolios ${\pi_1}^+$, ${\pi_1}^-$, ${\pi_1}^*$

$$\vec{\mathcal{V}}^+ = \overleftrightarrow{Tr}(0.9314, 0.9350, 0.9549, 0.9642),$$
$$\vec{\mathcal{V}}^- = \overleftrightarrow{Tr}(0.9784, 0.9771, 0.9608, 0.9441),$$
$$\vec{\mathcal{V}}^* = \overleftrightarrow{Tr}(0.9547, 0.9559, 0.9579, 0.9579).$$

The energy measure of EDF $\vec{\mathcal{V}}^*$ of portfolio π^* equals $d(\vec{\mathcal{V}}^*) = 0.02538$.

The entropy measure of EDF $\vec{\mathcal{V}}^*$ of portfolio π^* is $e(\vec{\mathcal{V}}^*) = 0.000248$.

Linear portfolio analysis is not possible for considered portfolio ${\pi_1}^*$.

CONCLUSIONS

1. The main objective of this paper was to compare the usefulness of the recommendations made by stockbrokers and suggested by the fuzzy portfolio analysis which was conducted with the use of fuzzy discount factor.
2. The results show that the fuzzy portfolio analysis was more accurate for the companies in construction sector assessed by a forecast of the closest change in prices than the recommendations of the brokers.
3. Using OFNs an initial decrease in the value of a portfolio was anticipated. The decrease occurred in both of analysed strategies, however, in case of the second strategy the decrease was of a smaller value.

CONCLUSIONS

4. The case study shows that recommendations obtained with the use of OFNs are better if we consider energy and entropy measures (their values are smaller). It can also result from the fact that OFN recommendation, unlike most of brokers' recommendations, are short-term recommendations.

5. The results based on OFNs are better because they are less imprecise.

THANK YOU FOR YOUR ATTENTION

ITISE 2022 (8th International conference on Time Series and Forecasting), 27th-30th June, Gran Canaria, Spain

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THANK YOU FOR YOUR ATTENTION

ITISE 2022 (8th International conference on Time Series and Forecasting), 27th-30th June, Gran Canaria, Spain