

Forecasting of COVID-19 Hospital Occupancy Using Differential Flatness



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Forecasts Based on Epidemiologic Models

Challenges in forecasting:

- Standard models suffer from various limitations
 - States of the system are generally unknown
 - Sensitive to initial conditions
- Dynamics driven by people's behavior and governmental interventions (exogenous drivers)
- Hospital occupancy main decision-factor

New flatness-based approach:

- Differentially flat model inputs delivers leading indicators for the epidemic
- Direct calculation of model states using only the infection numbers
- Reliable prediction of hospital occupancy

Coronavirus – “Worst Case Scenario”

Assumption: 60-70% of the population affected

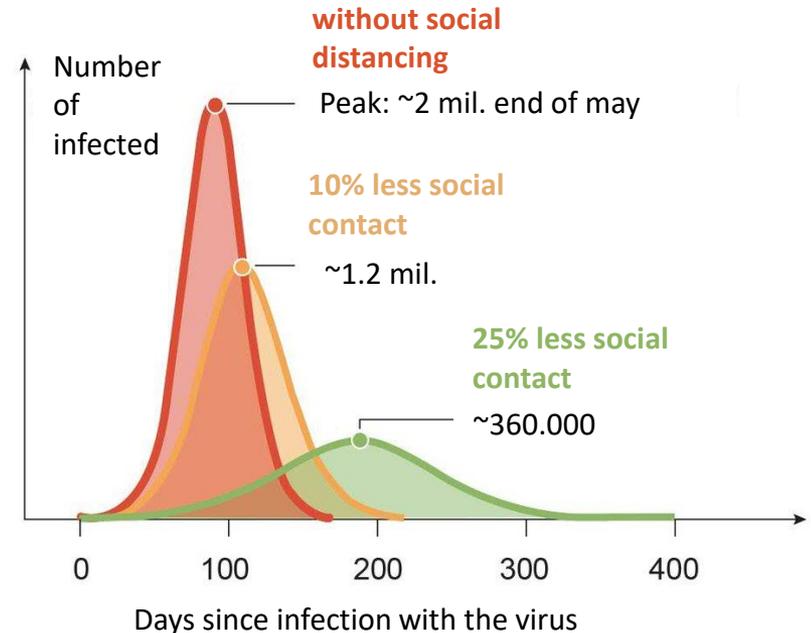


Figure: © APA, source: TU Wien/dwh, March 2020

The SIR Pandemic Model: Accounting for Interventions

$$\frac{dS}{dt} = -\frac{\beta IS}{N} + u$$

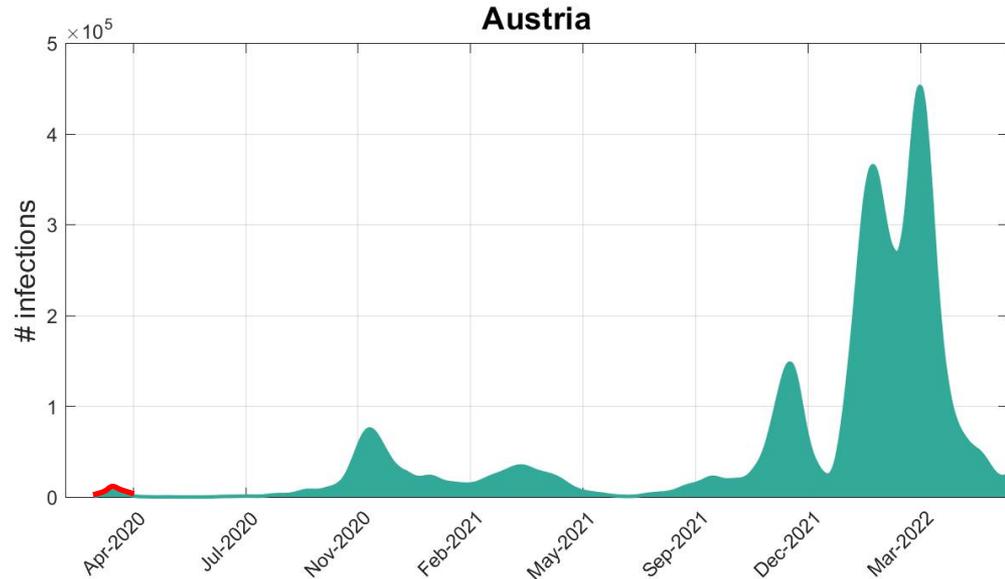
$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I$$

$$S(0) = S_0$$

$$I(0) = I_0$$

$S(t)$... number of susceptibles

$I(t)$... number of infected



The standard SIR-model cannot model changing interventions, people's behavior and multiple epidemic waves.

Add additional exogenous input u in order to describe the dynamics accurately.

SIR-Model with Exogenous Input

$$\frac{dS}{dt} = -\frac{\beta IS}{N} + u(t)$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I$$



differential flatness

$$u(t) = \frac{N}{\beta} \left(\frac{\ddot{I}}{I} - \frac{\dot{I}^2}{I^2} \right) + \dot{I} + \gamma I$$

$$S(t) = \frac{\dot{I} + \gamma I}{\frac{\beta}{N} I}$$

States x and input u can be determined **directly** from the **output (measured number of infected) and its derivatives**.

$$\mathbf{x}(t) = \Psi_x(y_1(t), \dot{y}_1(t), \dots, y_1^{(\delta_1)}(t), y_2(t), \dots)$$

$$\mathbf{u}(t) = \Psi_u(y_1(t), \dot{y}_1(t), \dots, y_1^{(\delta_1)}(t), y_2(t), \dots)$$

Isidori, Alberto, E. D. Sontag, and M. Thoma: *Nonlinear control systems*. Vol. 3. London: Springer, 1995.

M. Fliess, J. L. Lévine, P. Martin and P. Rouchon: *Flatness and defect of non-linear systems: introductory theory and examples*. International Journal of Control 61(6), pp. 1327-1361, 1995.

SIR-Model with Exogenous Input

$$\frac{dS}{dt} = -\frac{\beta IS}{N} + u(t)$$
$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I$$

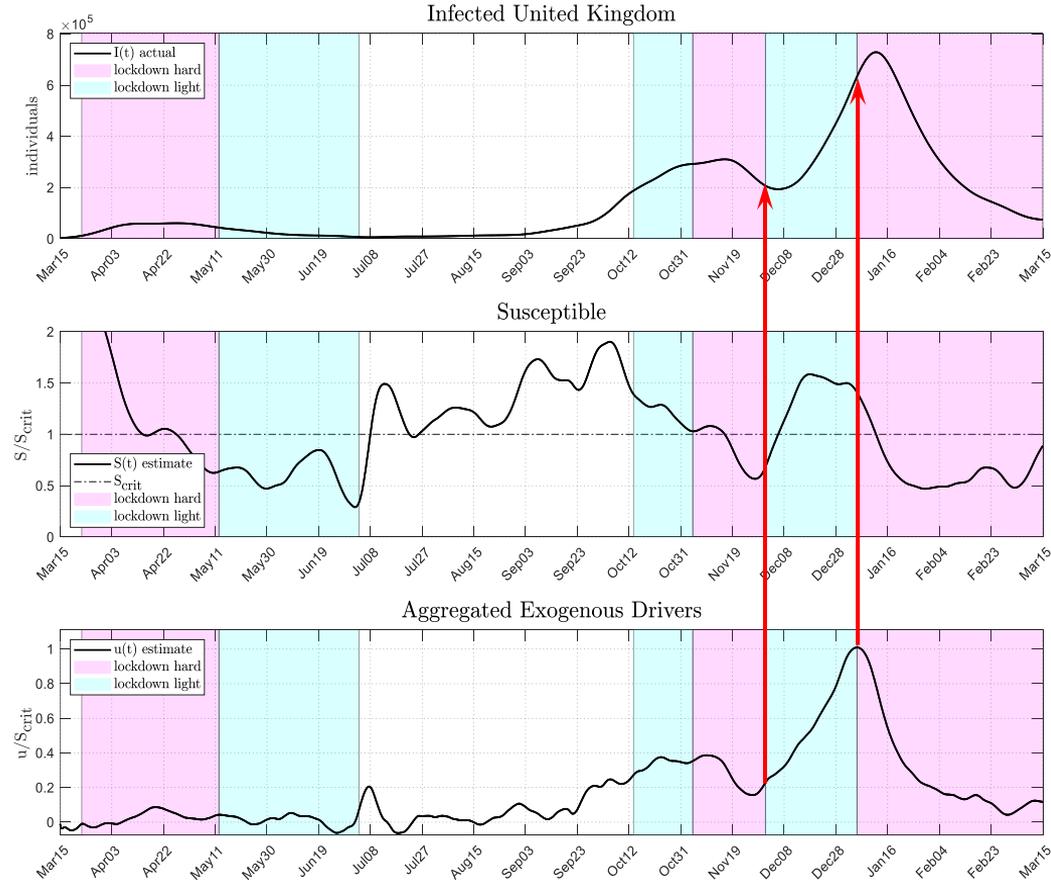
↓ differential flatness

$$u(t) = \frac{N}{\beta} \left(\frac{\ddot{I}}{I} - \frac{\dot{I}^2}{I^2} \right) + \dot{I} + \gamma I$$

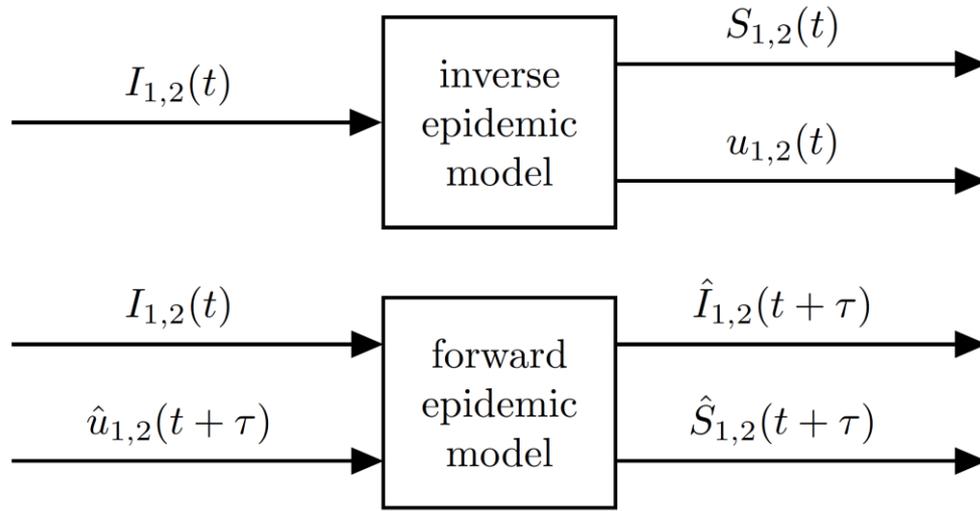
$$S(t) = \frac{\dot{I} + \gamma I}{\frac{\beta}{N} I}$$

Key take-aways:

$u(t)$ constitutes a **leading indicator** of the pandemic and determines its future evolution. Unmeasurable state $S(t)$ can be **estimated**.



From Analysis to Case Number Forecasts



Based on the differential flatness property, the inverse epidemic model provides **real-time estimates** of its **states** as well as its **exogenous inputs** that drive the dynamics.

Using **projections of the exogenous inputs**, the epidemic model provides forecasts of its future states.



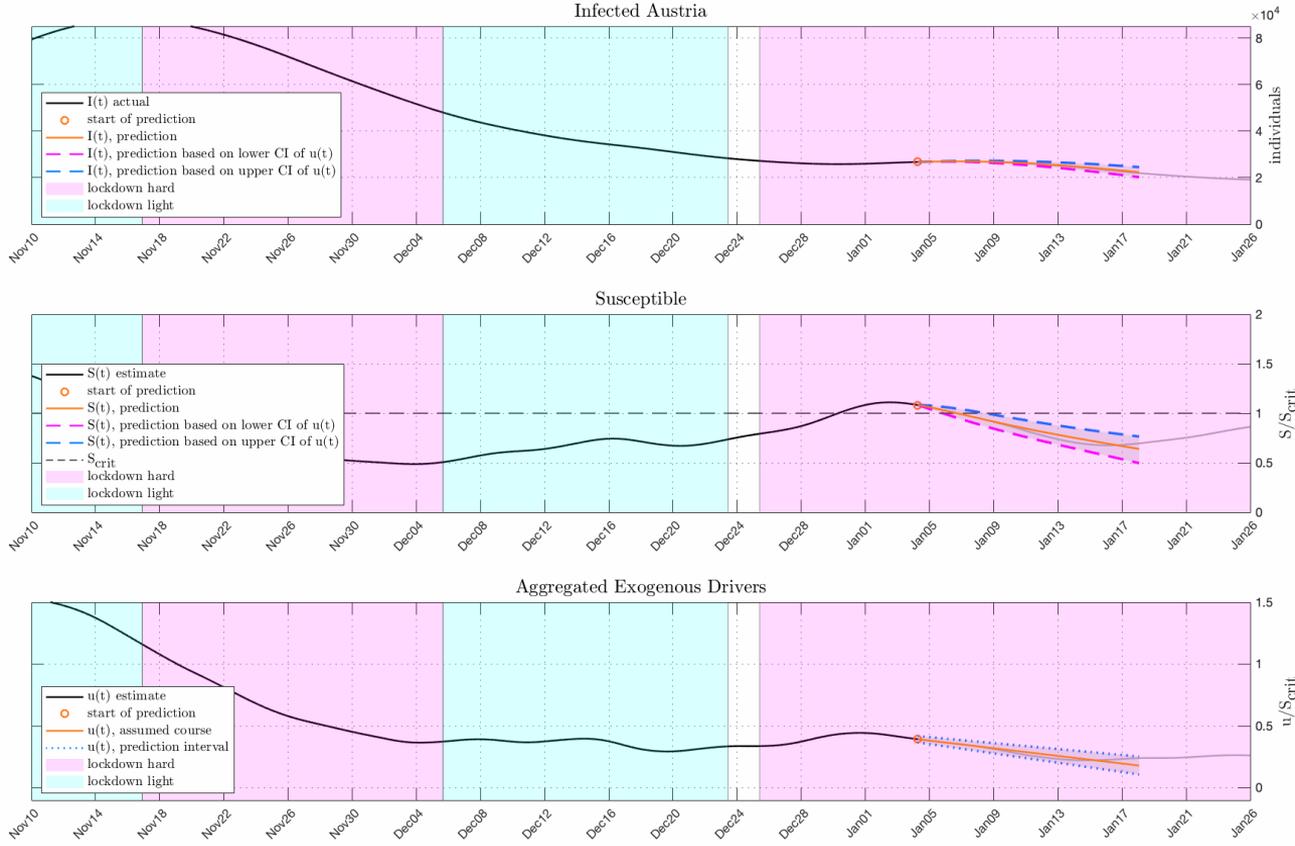
Key take-away:

Accuracy of case number forecasts is high because **exogenous drivers change very smoothly**. This makes their projections straight-forward.

Hametner et al.: *Estimation of exogenous drivers to predict COVID-19 pandemic using a method from nonlinear control theory*. Nonlinear Dynamics 106, pp. 1111-1125, 2021.



Three/Four-Week Forecasts: How Accurate Is Our Method?



Hospital Occupancy: Determining Dynamic Admission Rates

$$\chi(t) = \sum_{k=t-d_{max}}^t \varphi(t-k) r(k) \pi(k)$$

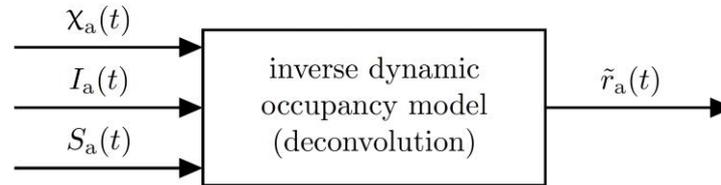
measured (green arrow pointing to $\chi(t)$)
estimated from data (blue arrow pointing to $\varphi(t-k)$)
unknown (red arrow pointing to $r(k)$)
modeled (blue arrow pointing to $\pi(k)$)

$\chi(t)$... occupied hospital or ICU beds on day t

$\varphi(t-k)$... convolution kernel, i.e. conditional probability of stay in hospital or ICU

$\pi(k) = \pi(S(k), I(k))$... Number of newly infected individuals on day k

$r(k)$... admission rate (probability of requiring hospital care after infection)



Key take-aways:



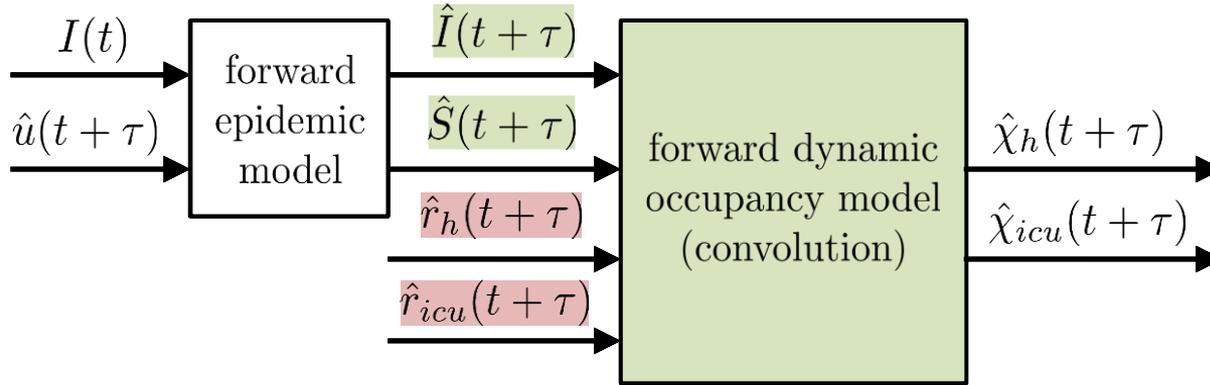
The case-specific admission rates $r_a(t)$ are **strongly time variant**.

We **estimate admission rates in real-time** from an inverse dynamic occupancy model.

Hametner et al.: *Intensive care unit occupancy predictions in the COVID-19 pandemic based on age-structured modelling and differential flatness*. Nonlinear Dynamics, 2022.



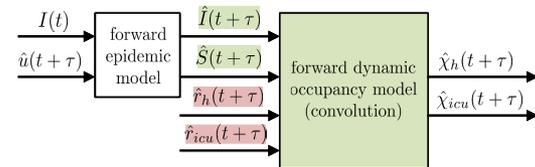
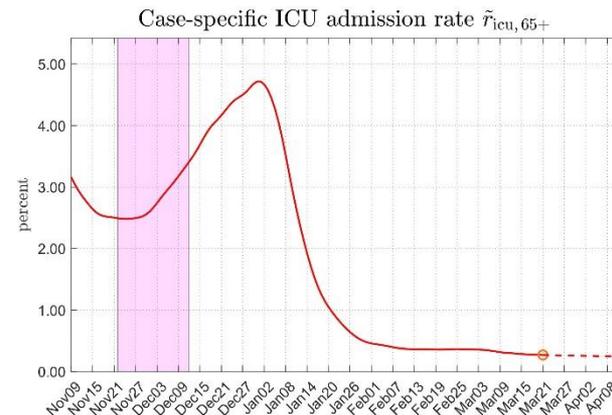
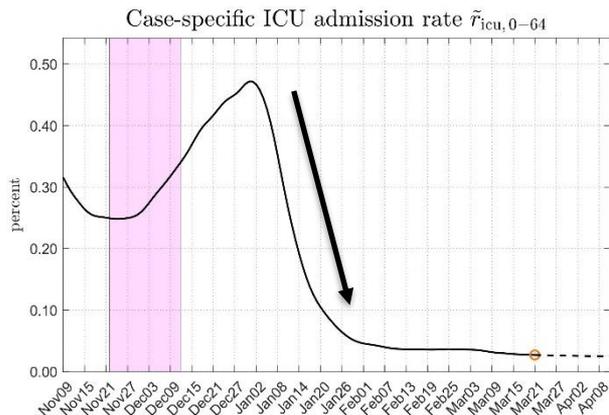
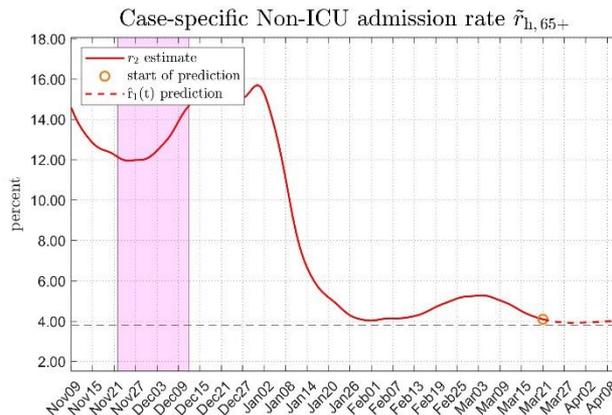
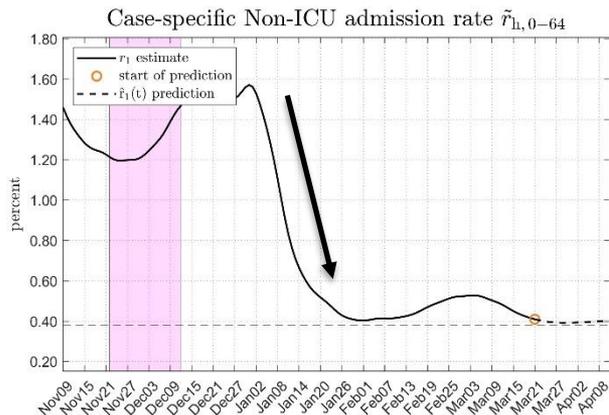
Analyzing and Predicting Hospital Occupancy



Key take-away:

The predicted admission rates and number of infected deliver the desired **occupancy forecast for intensive and normal care units.**

Admission Rates for Normal and Intensive Care per Age Group



Observations:

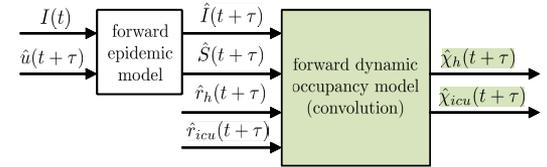
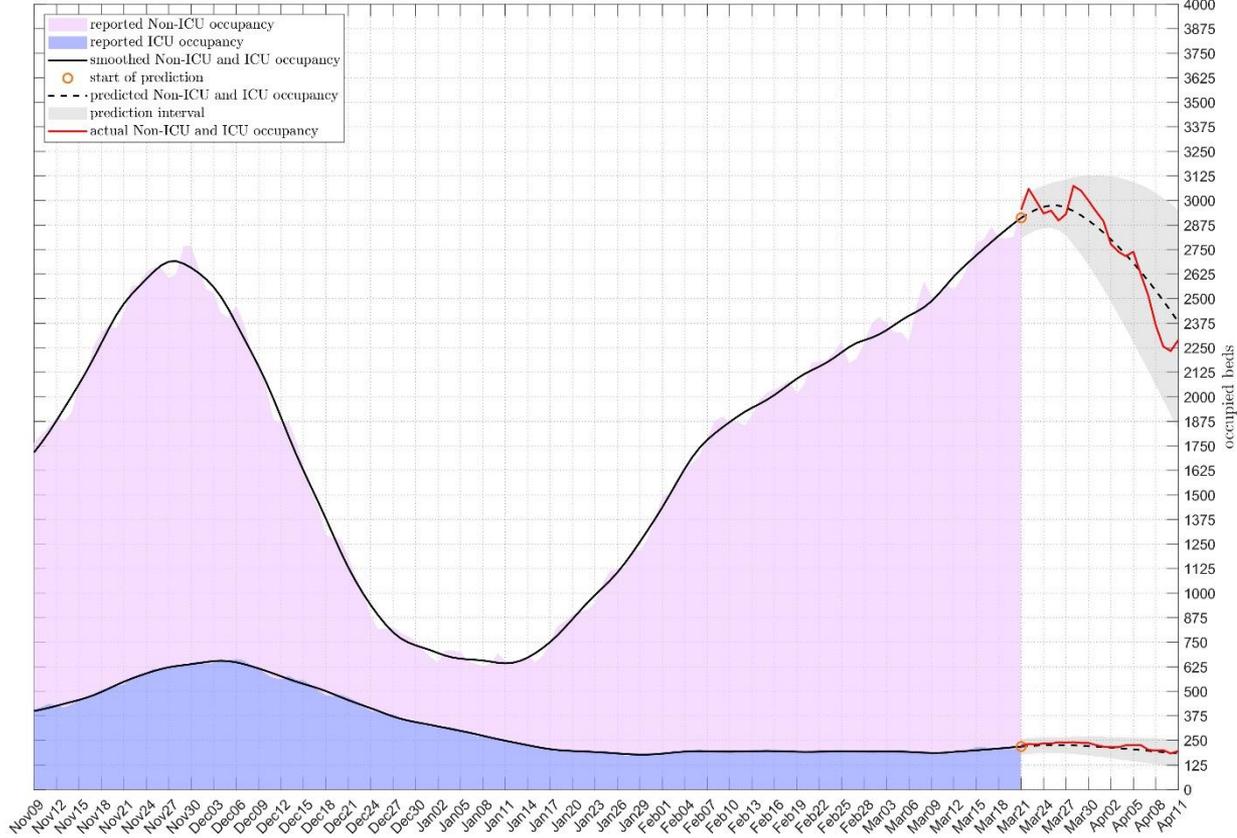
- In January omicron became the dominant variant
- Admission rate greatly decreased

Prediction:

- Almost stationary admission rates

Three-Week Forecasting Hospital Occupancy in Austria

Non-ICU and ICU occupancy, 3-week forecast

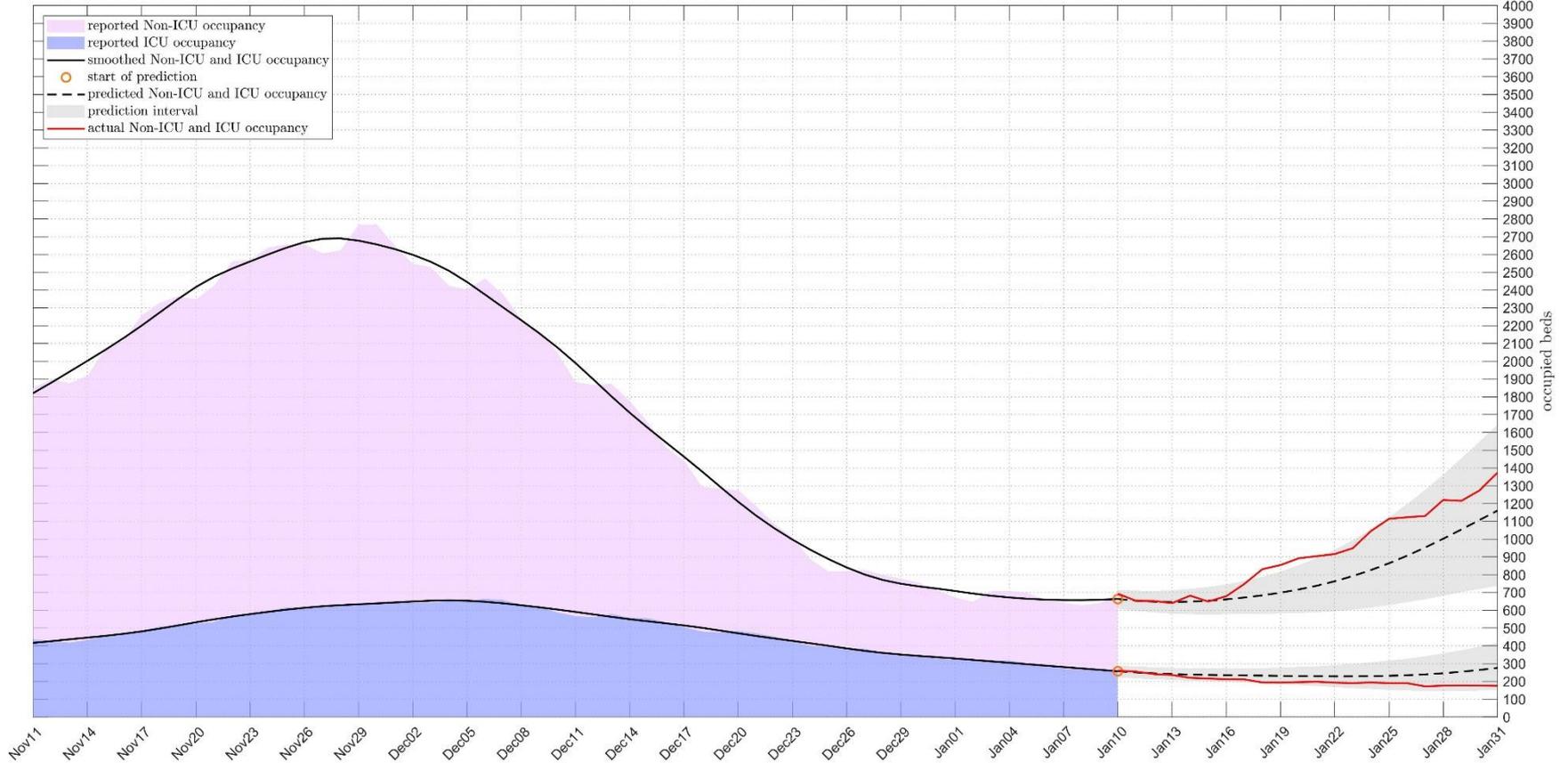


Observations:

- Very high hospital occupancy
- Strong daily fluctuations
- ICU occupancy is almost constant since February
- A turning point in the hospital occupancy is predicted

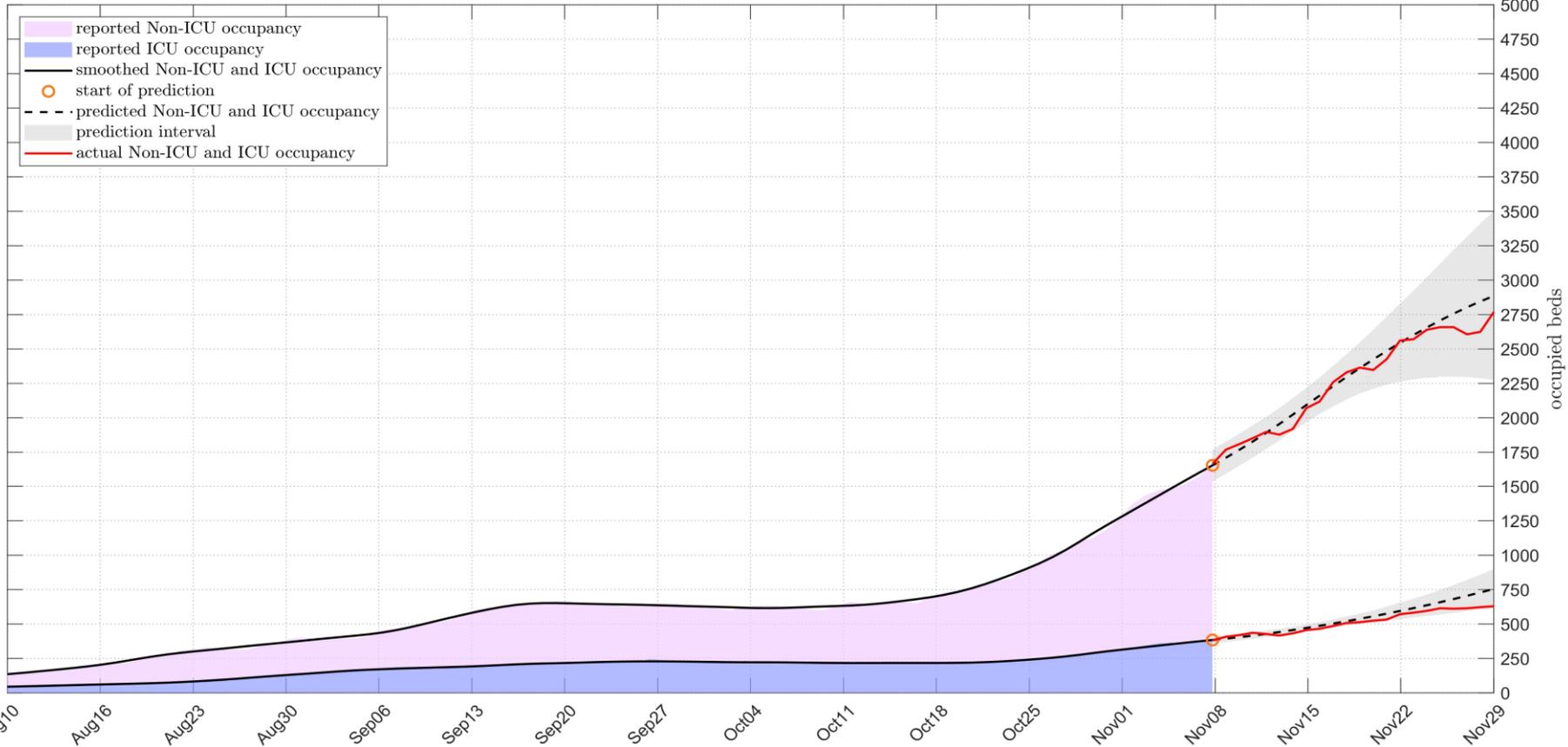
Three-Week Forecasting Hospital Occupancy in Austria

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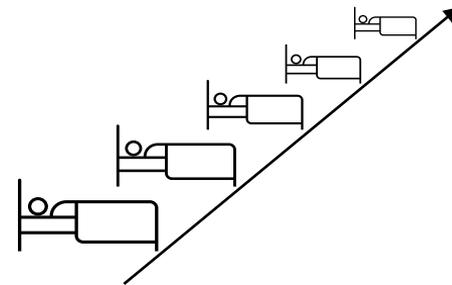
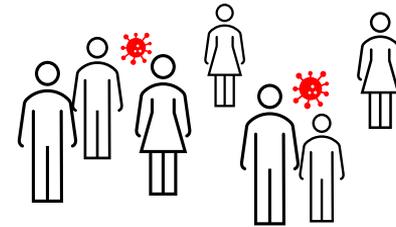
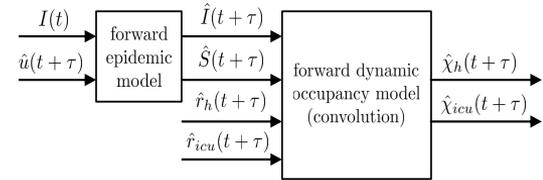
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Summary

- New method allows the extension of many known models which are differential flat
 - Introduced exogenous inputs are the leading indicators of the epidemic
 - Exogenous input has a physical meaning and can be easily interpreted
 - Smooth evolution of these exogenous drivers lead to reliable predictions
- From hospital data the time-varying admission rates are calculated
- Hospital occupancy forecast is obtained from predicted admission rates and number of infected



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