Performance evaluation of a family of GARCH processes based on Value-at-risk forecasts

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1 Motivation

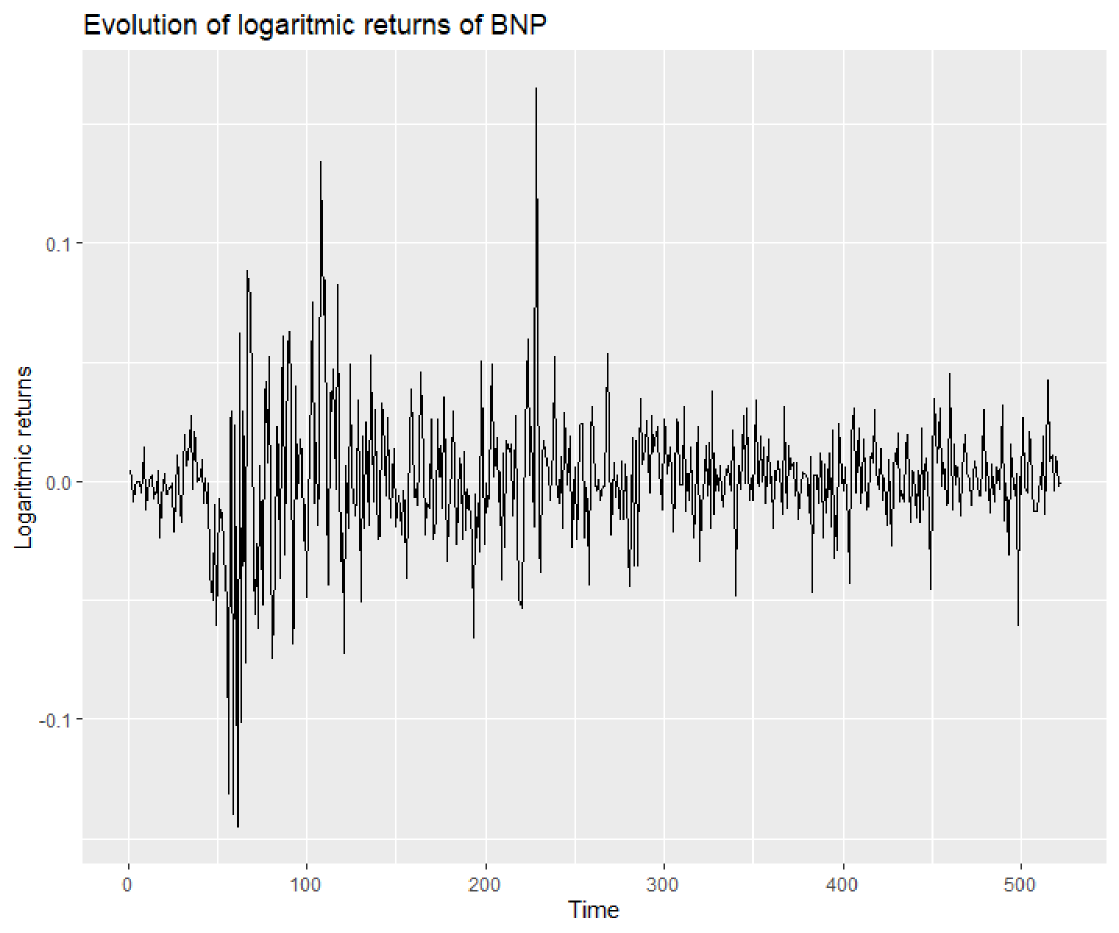
- Imagine you want to invest into a new asset that is traded for a short period of time, and you are interested in risk evaluation (Value at risk for example)
- You don’t have enough historical data to backtest Value at risk estimate
- Best approach is to take a model that works best for majority of similar data
- Let’s say we are interested in a new stock on European stock market
- We run analysis on the biggest European stocks (46) that have a sufficient long historic data available
- The models that are efficient will probably be suitable for modeling and assessing risk associated with the new stock as well

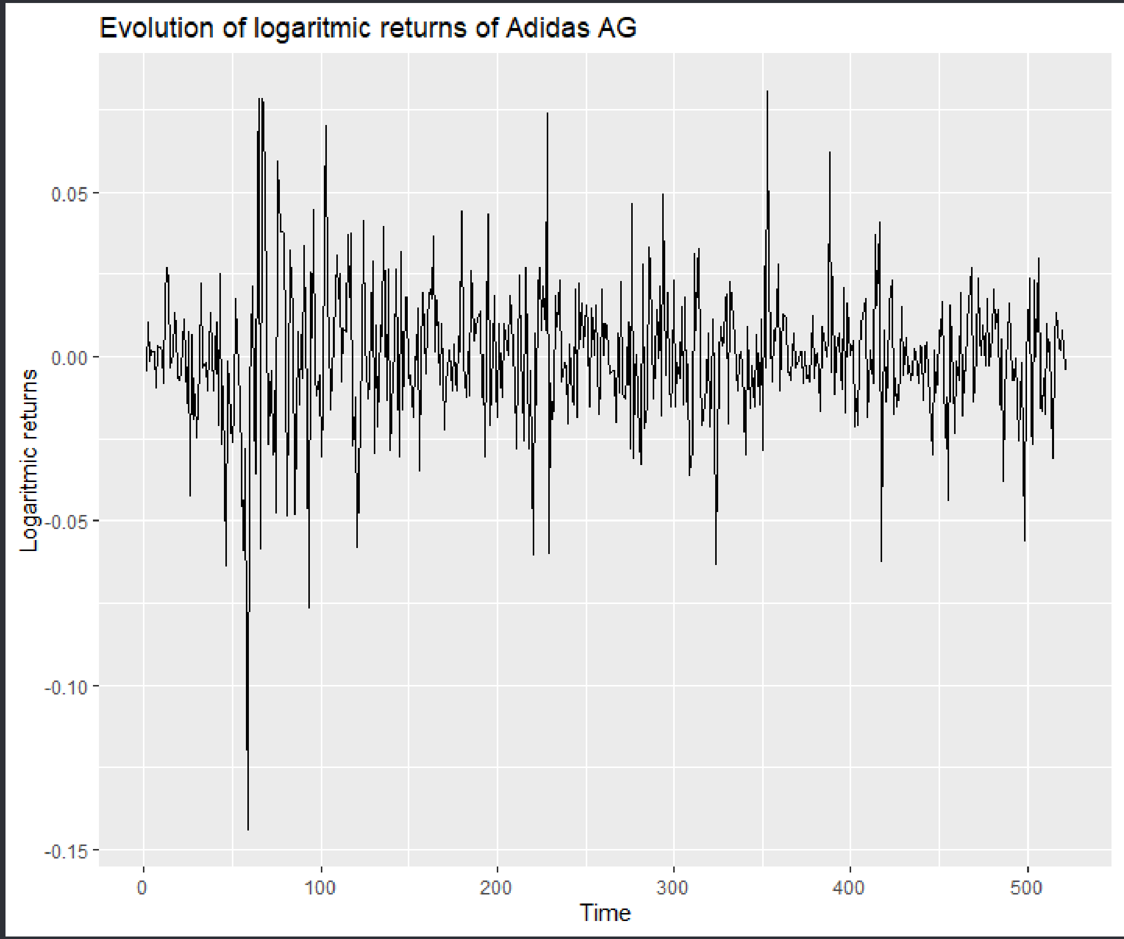
- How to decide which models are efficient and which are not?

2

Modeling of stock returns

- One of the main features associated with financial time series is a non-constant volatility
  - Best practice is to model data by a family of GARCH processes





- Let $r_t = ln\left(\frac{P_t}{P_{t-1}}\right)$ be logarithmic return at time t (either profit or loss)
- We assume following dynamics for development of returns

$$r_t = \mu + \sigma_t z_t$$

$$z_t \text{ sú i.i.d}, \qquad z_t \sim \mathrm{D}(0,1)$$

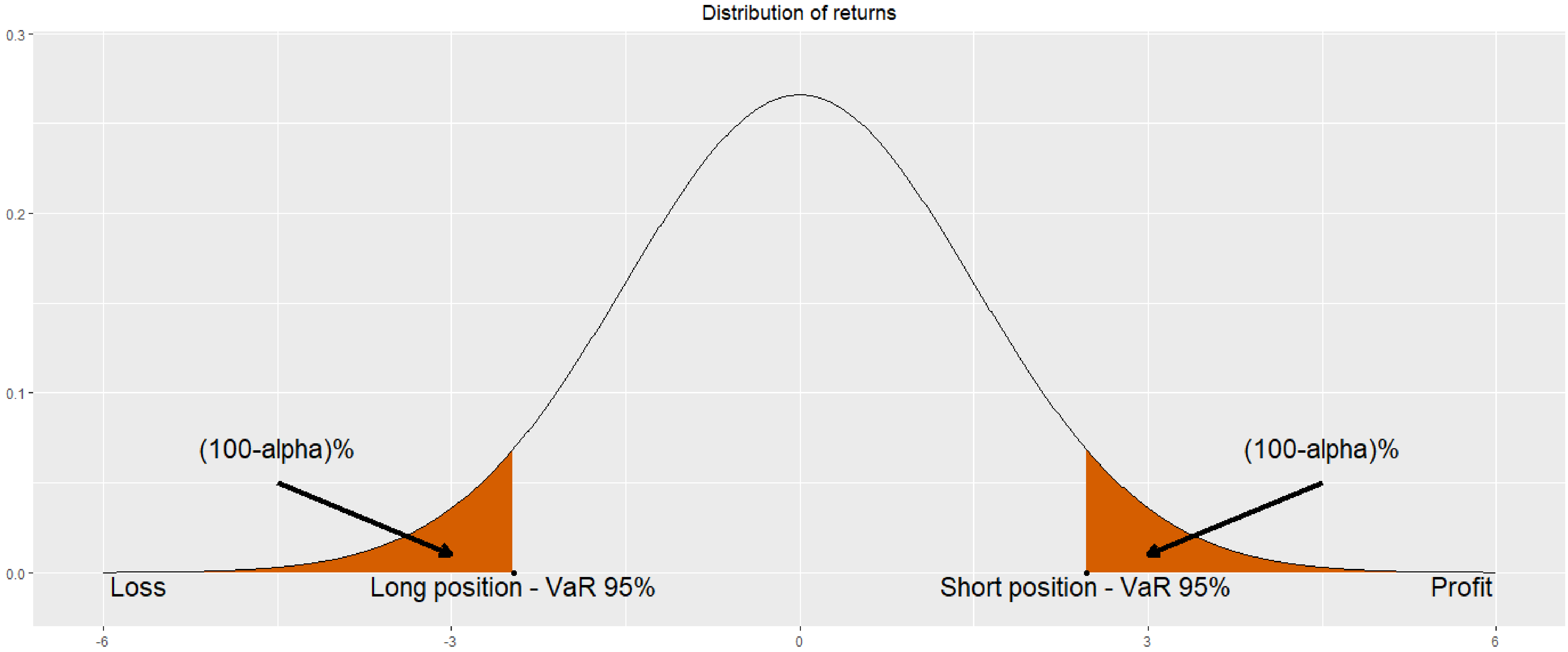
- Normal, student t, skewed normal and skewed student t distribution
- GARCH processes:
  1. Standard GARCH(1,1)
  2. To capture persistence of volatility, meaning shocks today will have significant impact on all future values of volatility– IGARCH(1,1), FIGARCH(1,1), component GARCH(1,1)
  3. To capture asymmetric response to negative and positive innovations, or so-called leverage effect – TGARCH(1,1), GJR-GARCH(1,1), EGARCH(1,1)

- Final models will be a combination of one of the distribution and one of the GARCH processes, in total 28 models

Value-at-Risk

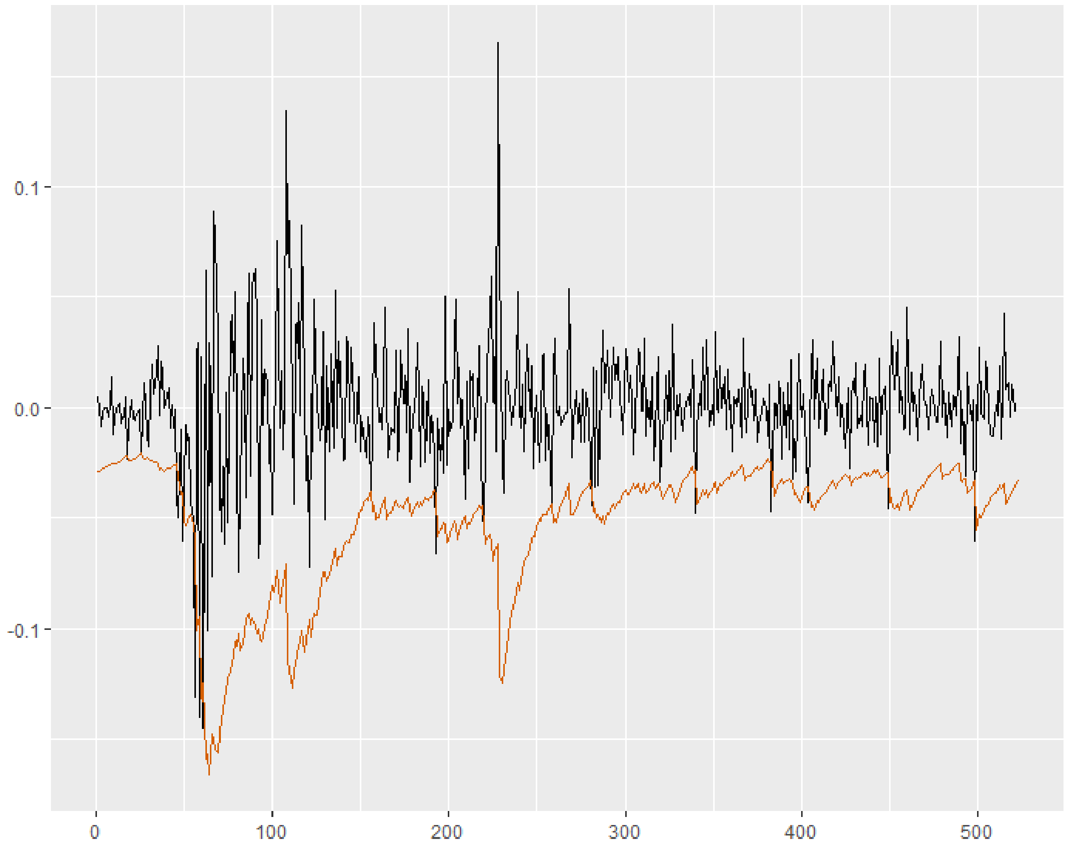
- Risk measure associated with economic losses on financial markets
- Defined as the loss observed during an N day period that at a confidence level α will not be exceeded
- Can be applied to any arbitrary portfolio and time period
- Our analysis
    - Time period equal to 1 day
    - Only interested in risk associated with one stock at the time
    - α equal to $99\%, 97.5\%, 95\%$

$$VaR_\alpha = inf\left(x | P(r_t < x) \leq (1 - \alpha)\right)$$

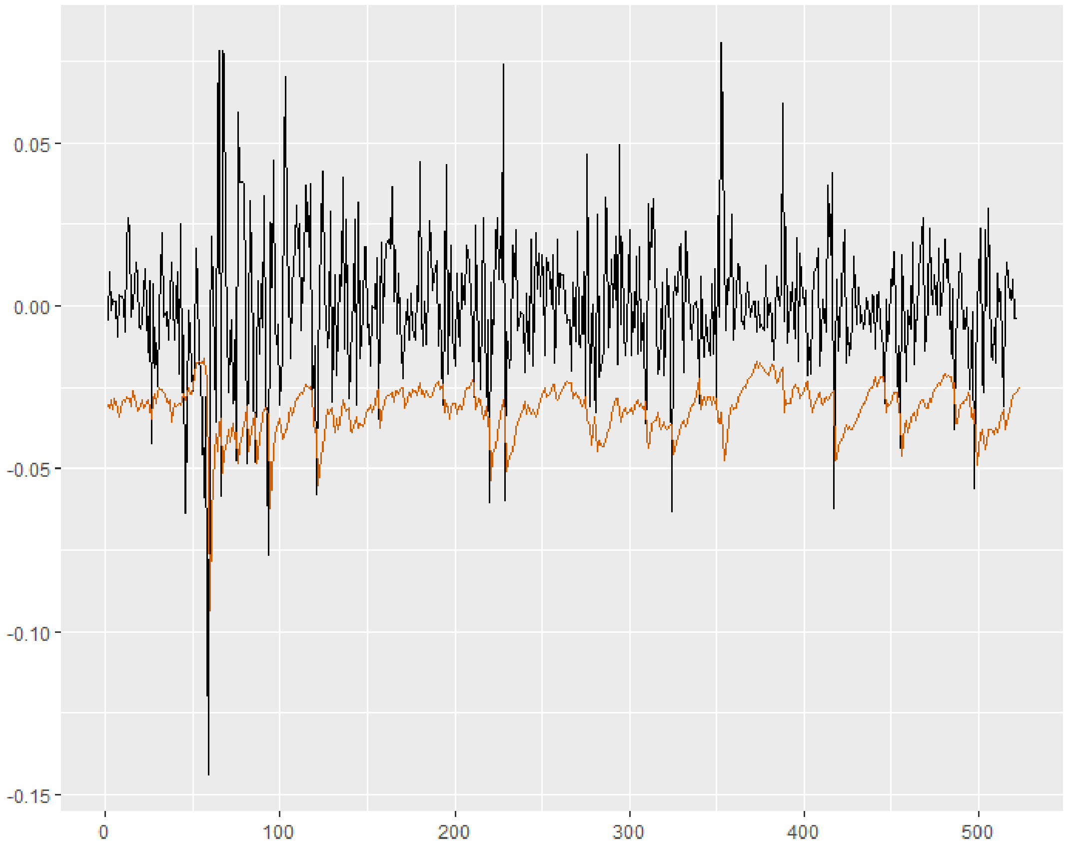


Distribution of returns
0.3
0.2
0.1
0.0
(100-alpha)%
(100-alpha)%
Loss
Long position - VaR 95%
Short position - VaR 95%
Profit
-6
-3
0
3
6

Logaritmic returns of BNP and VAR_97.5% - iGARCH + skewed student t distribution



Logaritmic returns of Adidas AG and VAR_95% - eGARCH + Normal distribution



3 Backtesting of Value at Risk

Statistical tests

- Use of the past values to test whether value at risk prediction are correct
- Kupiec's test (also called Unconditional coverage test)
  - test is focused on examining whether the frequency of exceeding the Value at risk boundary corresponds to a given confidence level α
- Independence test
  - tests whether the probability of occurrence or non-occurrence of exceedance on a given day is independent of what occurred the previous day
- Christoffersen's test (also called Conditional coverage test)
  - tests both hypothesis simultaneously

Loss function

- Tools to measure size of uncovered losses (losses beyond Value at risk)
- Based on penalizing models proportionally to some measure of distance between forecasted Value at risk and observed loss
- Caporin

$$L_t(r_t, VaR_t(\alpha)) = \begin{cases} \dfrac{(|VaR_t(\alpha)| - |r_t|)^2}{|VaR_t(\alpha)|}, & if\ r_t \leq VaR_t(\alpha) \\ 0, & else \end{cases}$$

- Proportional to Value at risk, allowing us to compare results on multiple different time series
- Quadratic - bigger differences are penalized more

4

Evaluation of model

DEA

- Data envelopment analysis (DEA) is a part of multi-criteria decision-making techniques
- Based on idea that units under investigation consume inputs and produce outputs
- We are working with undesirable outputs and desirable outputs and without any inputs

- Output of DEA is subset of efficient units and subset of inefficient units

Russelov model – SDP reformulation

- Non-radial DEA model
- Objective function is linear in unknown variables
- Desirable outputs – number of times model passed all tree statistical tests for specific time series for each confidence levels α considered (3)
- Undesirable outputs – mean value of loss function per time series for each confidence levels α considered (3)
- Mathematical program needs to be run for each unit under investigation separately

$$\min_{\phi_r,\psi_r,\theta_i,\lambda_j} \frac{1}{s+m}\left(\sum_{i=1}^{m}\theta_i + \sum_{r=1}^{s}\psi_r\right)$$

$$s.t. \quad \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta_i x_{io}, \quad i = 1,\dots,m$$

$$\sum_{j=1}^{n}\lambda_j y_{rj} \ge \phi_r y_{ro}, \quad r = 1,\dots,s$$

$$\sum_{j=1}^{n}\lambda_j = 1,$$

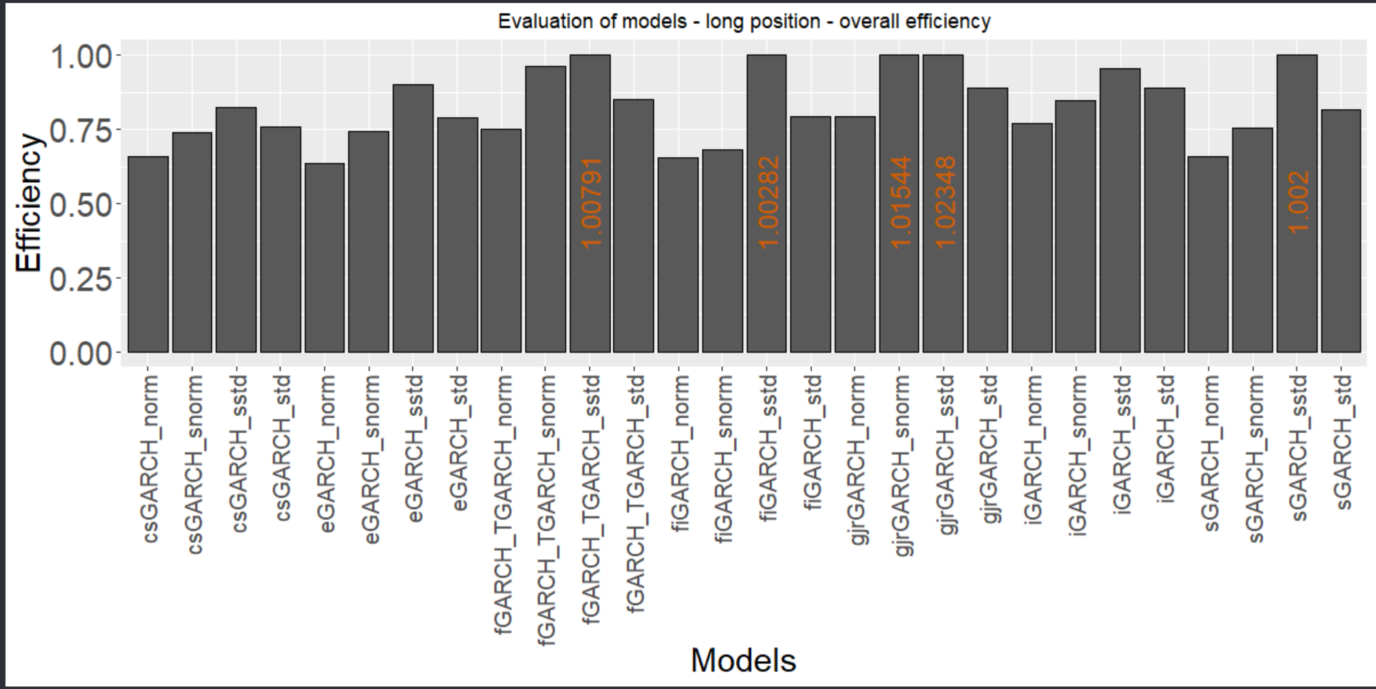
$$\lambda_j \ge 0 \quad j = 1,\dots,n,$$

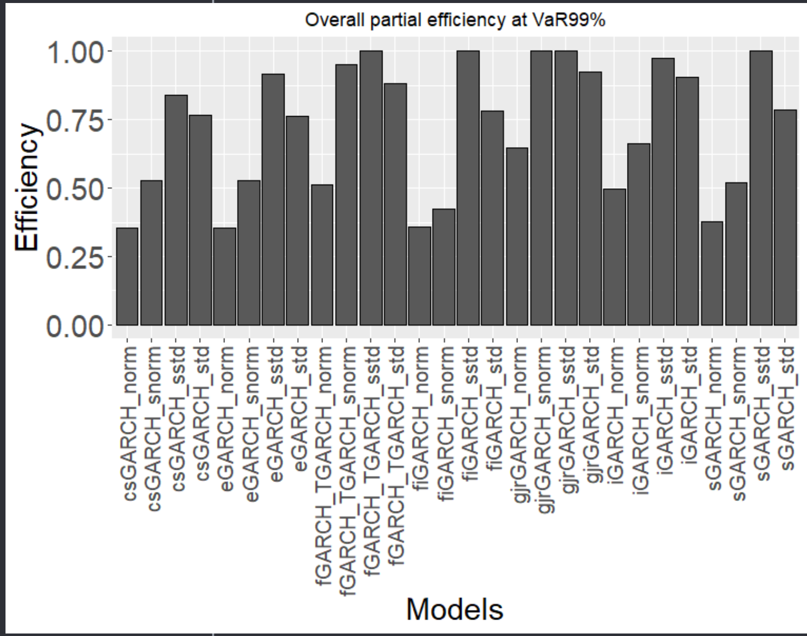
$$\theta_i \le 1 \quad i = 1,\dots,m,$$

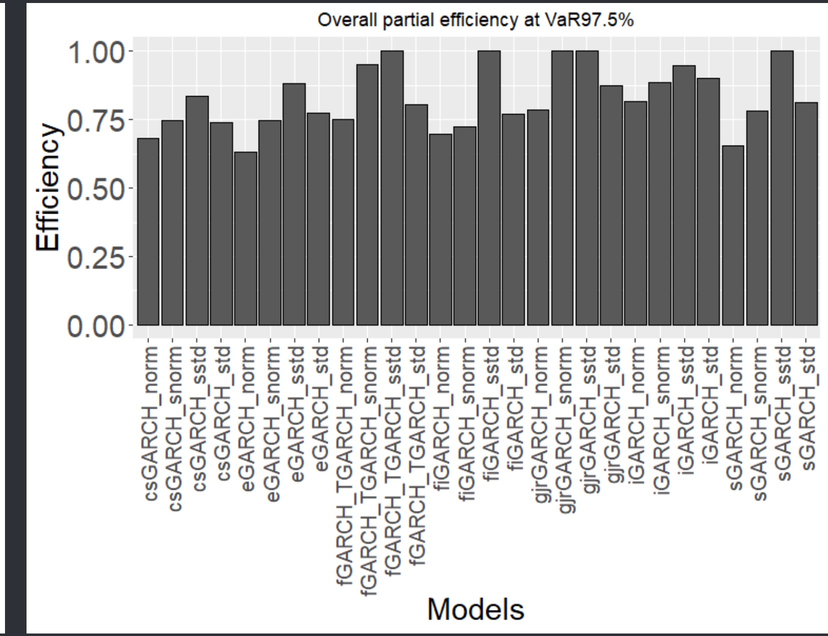
$$\phi_r \ge 1 \quad r = 1,\dots,s,$$

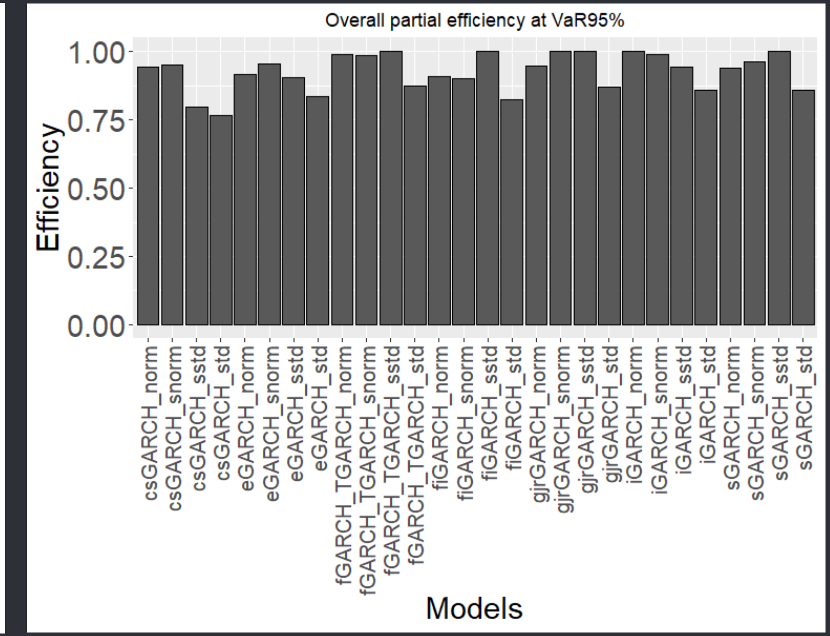
$$\begin{pmatrix} diag(\phi_1,\dots,\phi_s) & I_{s,s} \\ I_{s,s} & diag(\psi_1,\dots,\psi_s) \end{pmatrix} \succcurlyeq 0$$

5 Results


Evaluation of models - long position - overall efficiency
Efficiency
Models
1.00791
1.00282
1.01544
1.02348
1.002


Overall partial efficiency at VaR99%
Efficiency
Models


Overall partial efficiency at VaR97.5%
Efficiency
Models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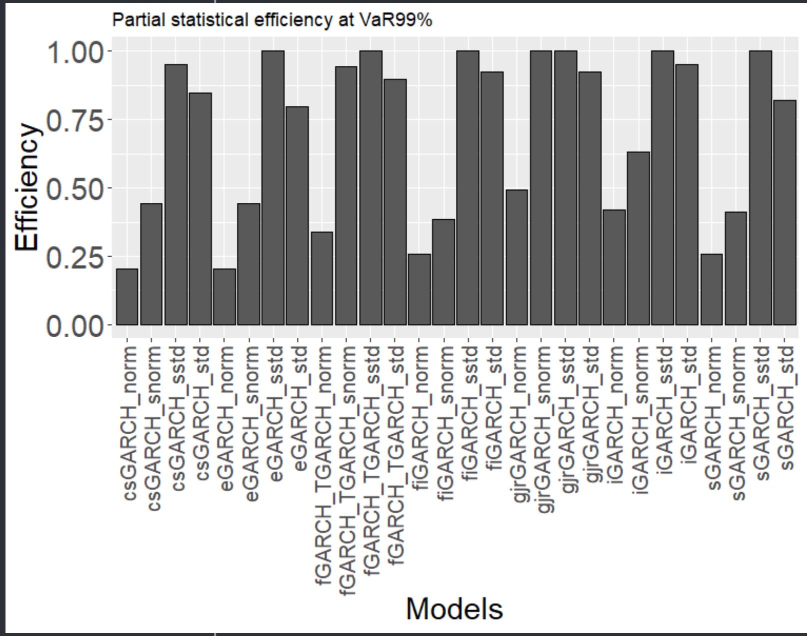
Partial statistical efficiency at VaR99%
Efficiency
1.00
0.75
0.50
0.25
0.00
csGARCH_norm
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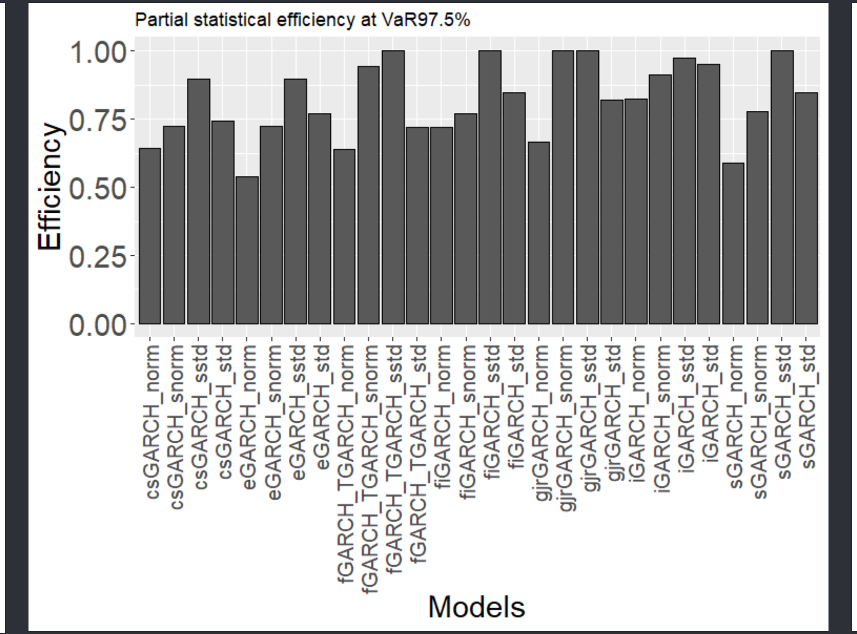
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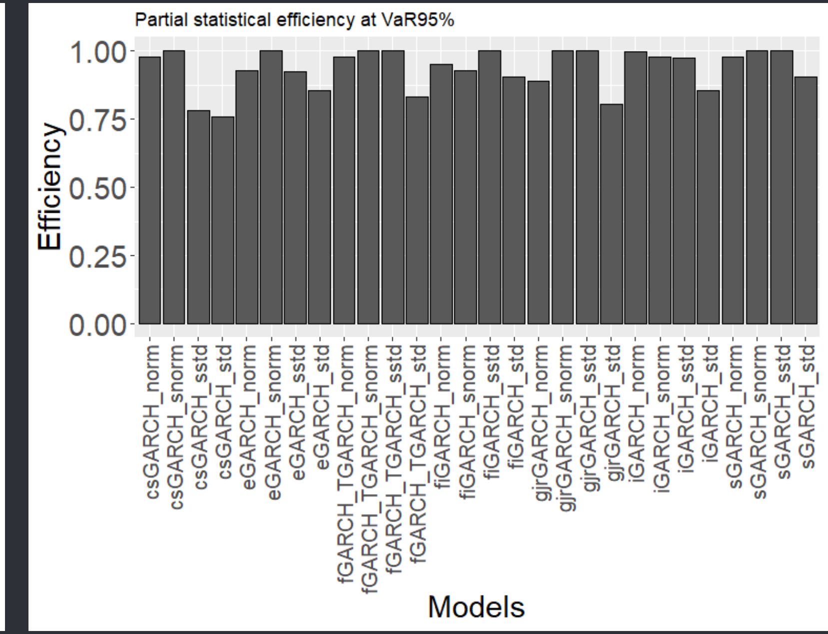
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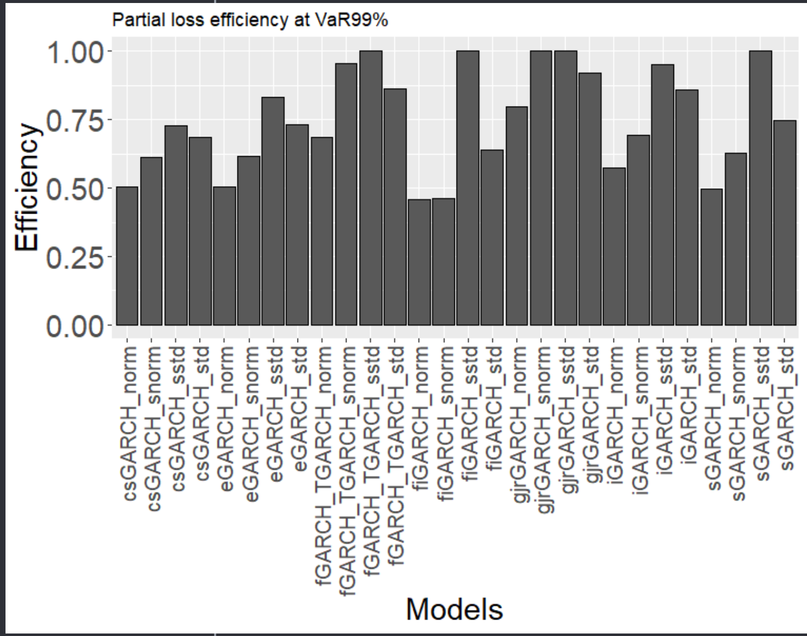
Partial loss efficiency at VaR99%
Efficiency
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0.75
0.50
0.25
0.00
csGARCH_norm
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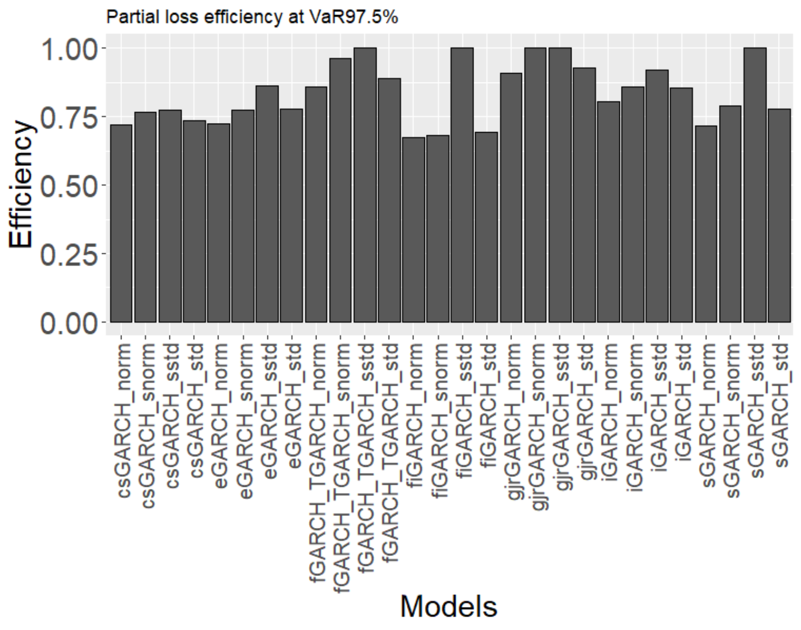
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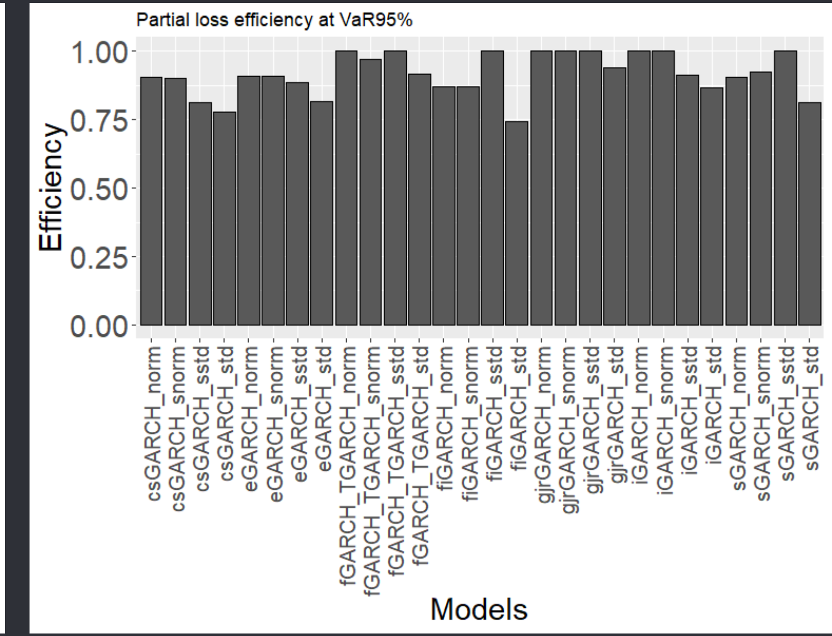
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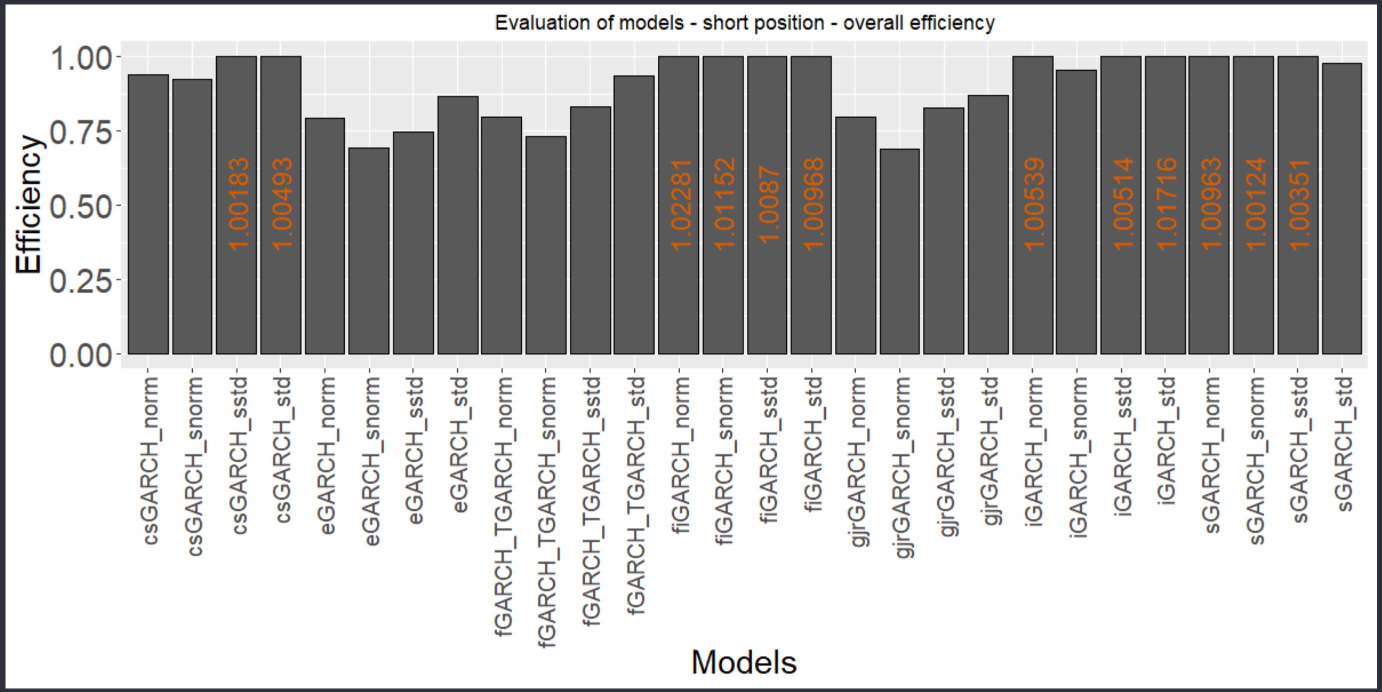
Evaluation of models - short position - overall efficiency
Efficiency
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0.75
0.50
0.25
0.00
1.00183
1.00493
1.02281
1.01152
1.0087
1.00968
1.00539
1.00514
1.01716
1.00963
1.00124
1.00351
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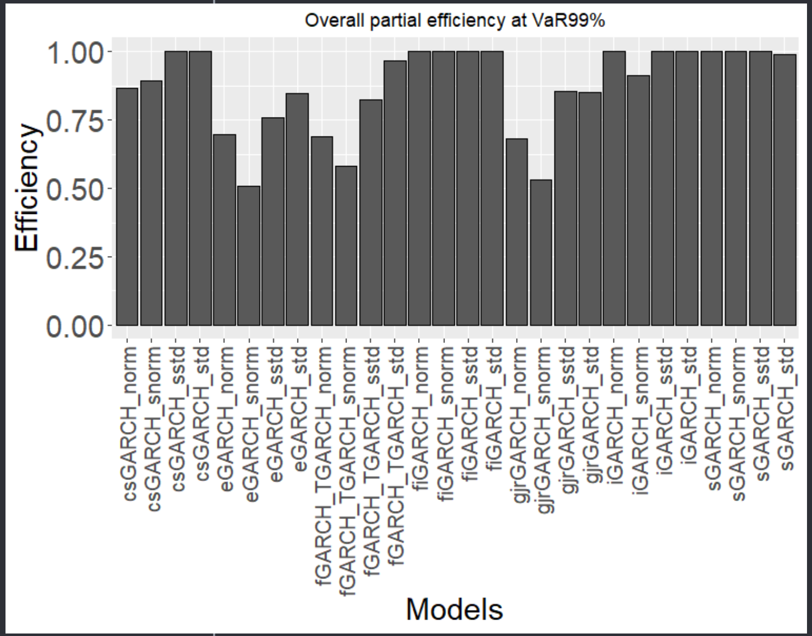
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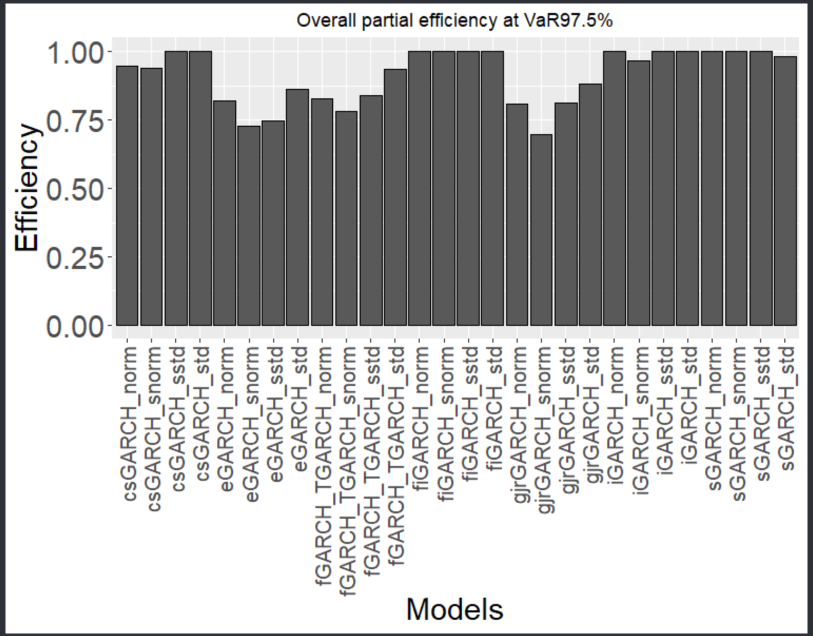
Overall partial efficiency at VaR97.5%
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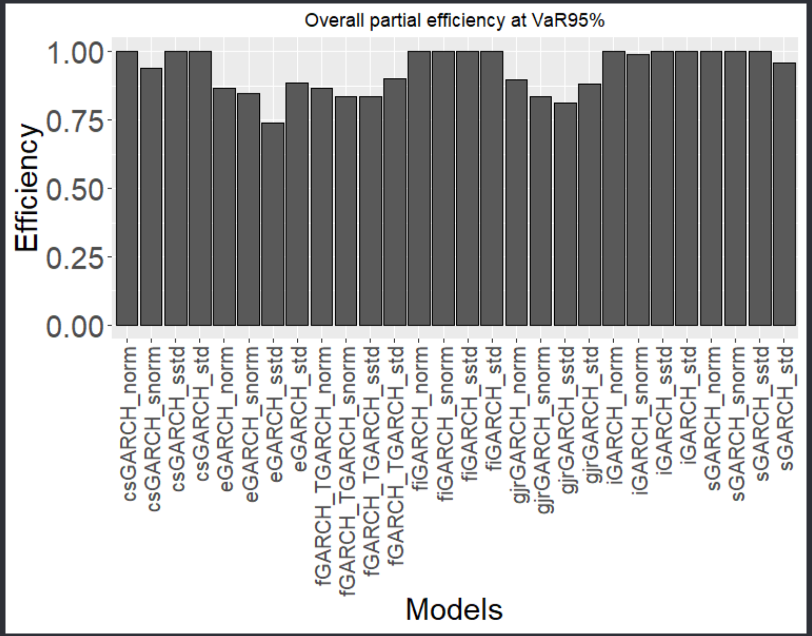
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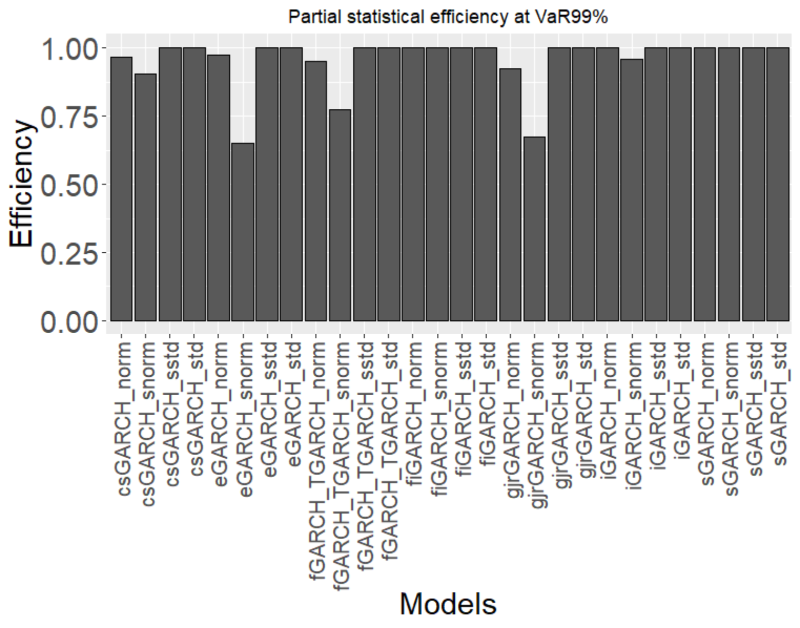
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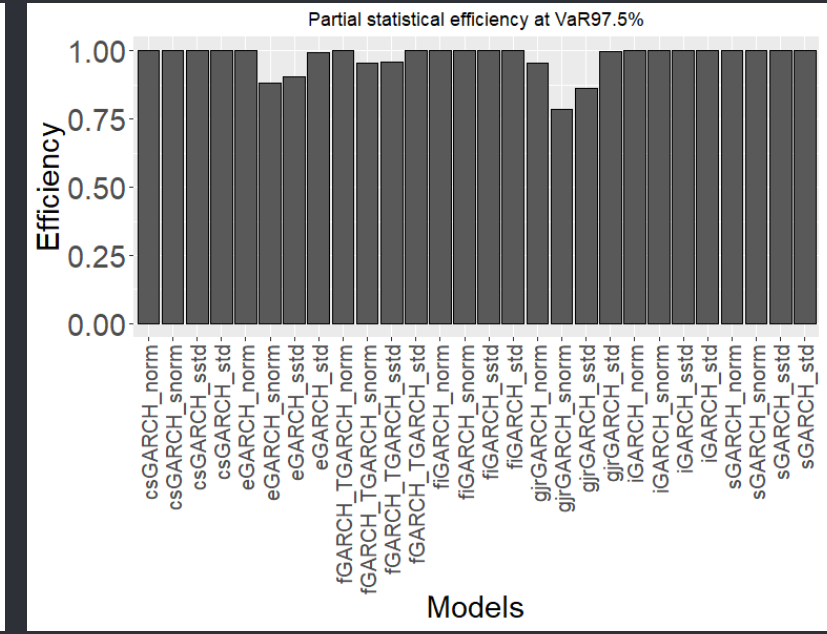
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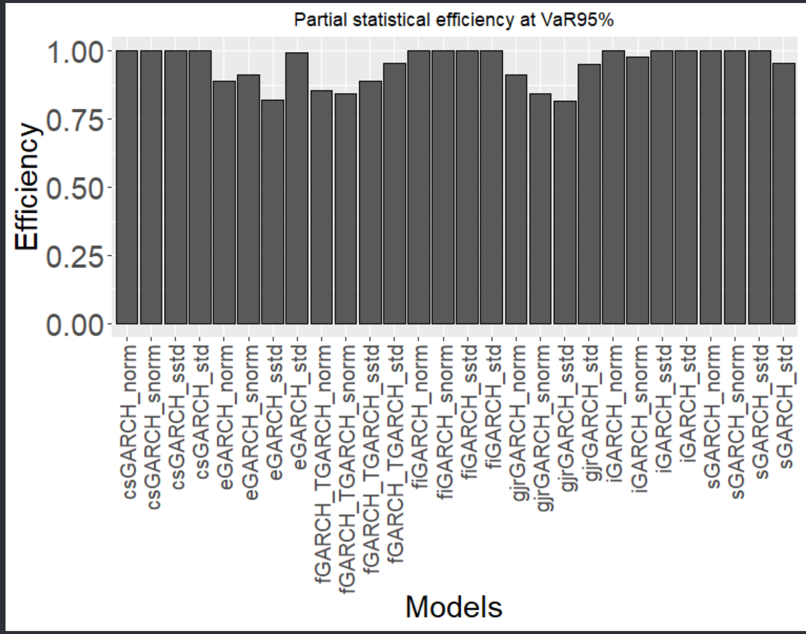
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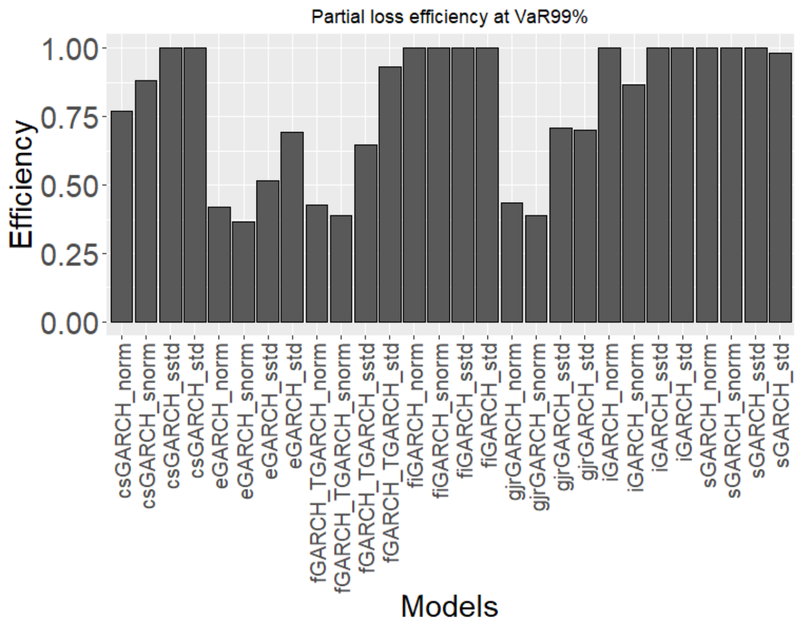
Partial loss efficiency at VaR99%
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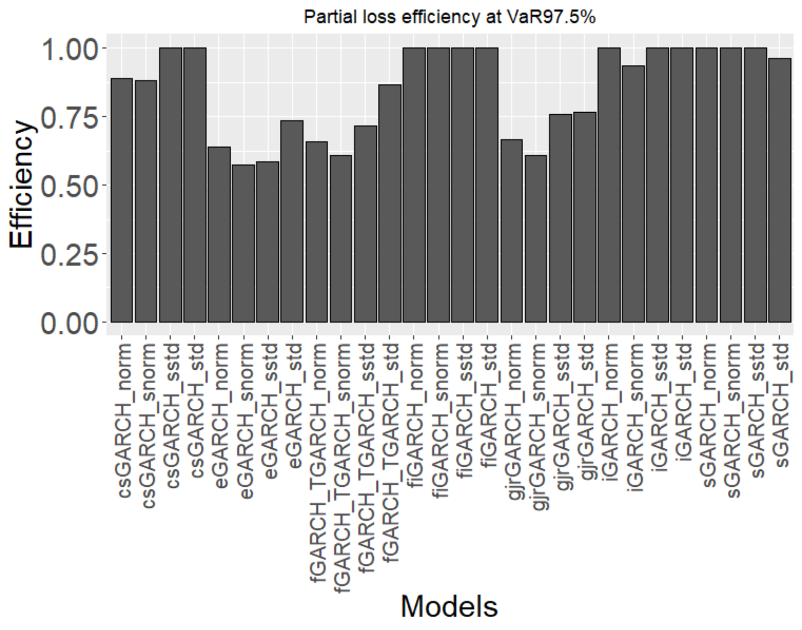
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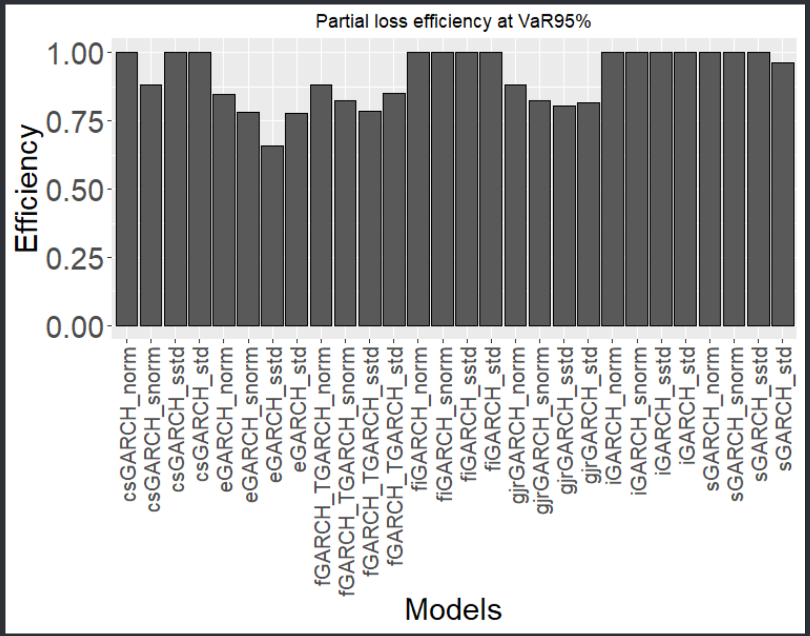
eGARCH_sstd
eGARCH_std
fGARCH_TGARCH_norm
fGARCH_TGARCH_snorm
fGARCH_TGARCH_sstd
fGARCH_TGARCH_std
fiGARCH_norm
fiGARCH_snorm
fiGARCH_sstd
fiGARCH_std
gjrGARCH_norm
gjrGARCH_snorm
gjrGARCH_sstd
gjrGARCH_std
iGARCH_norm
iGARCH_snorm
iGARCH_sstd
iGARCH_std
sGARCH_norm
sGARCH_snorm
sGARCH_sstd
sGARCH_std
Models


Partial loss efficiency at VaR95%
Efficiency
1.00
0.75
0.50
0.25
0.00
csGARCH_norm
csGARCH_snorm
csGARCH_sstd
csGARCH_std
eGARCH_norm
eGARCH_snorm
eGARCH_sstd
eGARCH_std
fGARCH_TGARCH_norm
fGARCH_TGARCH_snorm
fGARCH_TGARCH_sstd
fGARCH_TGARCH_std
fiGARCH_norm
fiGARCH_snorm
fiGARCH_sstd
fiGARCH_std
gjrGARCH_norm
gjrGARCH_snorm
gjrGARCH_sstd
gjrGARCH_std
iGARCH_norm
iGARCH_snorm
iGARCH_sstd
iGARCH_std
sGARCH_norm
sGARCH_snorm
sGARCH_sstd
sGARCH_std
Models

Thank you for paying attention