Trading on Short-term Path Forecasts of Intraday Electricity Prices

Tomasz Serafin\textsuperscript{1}, Grzegorz Marcjasz\textsuperscript{1}, Rafał Weron\textsuperscript{1}

\textsuperscript{1}Department of Operations Research and Business Intelligence, Faculty of Management, Wrocław University of Science and Technology, Poland
Modeling framework

- We consider transactions from the German continuous intraday market
- We look at the last three hours of trading before the delivery
- We consider ten 15-minute volume-weighed average prices
Forecasting methods

Point forecasts
- LASSO
- QR
- Gaussian Copula
- Random quantiles at $t_1$
- Direct
- LQC
- AQL

Probabilistic forecasts
- LASSO
- QR
- Multivariate t-Student
- Historical vectors of increments
- Historical vectors of errors

Path forecasts
- Multivariate t-Student
- Historical vectors of increments
- Historical vectors of errors

Starting point for paths
- VWAP at $t_0$
- VWAP at $t_0$
- Direct
- Direct
- Direct
- Direct
- Naive

Prediction band
- Similar Day (SD)
- LASSO bootstrap
- LASSO point
- Market data
Prediction bands

A simultaneous prediction band with coverage probability $1 - \alpha$ is a set of points $(B_{d,h,t_1}^{\text{up}}, \ldots, B_{d,h,t_{10}}^{\text{up}})$ that satisfies the following condition:

$$\mathbb{P} \left( X_{d,h,t_j} \leq B_{d,h,t_j}^{\text{up}}, \forall t_j \right) = 1 - \alpha,$$

where $j = 1, \ldots, 10$.
Prediction bands

- Prediction bands may be obtained in three different ways:
  - Directly from a number of generated path forecasts (Direct)
  - Using the approximate method based on forecasted quantiles (AQL)
  - Using the naive construction based on point forecasts (Naive)
Prediction bands: the direct approach

Point forecasts:
- LASSO
- QR
- Gaussian Copula
- Random quantiles at $t_1$
- Direct

Probabilistic forecasts:
- LASSO
- QR

Path forecasts:
- Multivariate t-Student
- Historical vectors of increments
- Historical vectors of errors

Starting point for paths:
- VWAP at $t_0$
- VWAP at $t_0$
- VWAP at $t_0$
- VWAP at $t_0$
- VWAP at $t_0$

Prediction band:
- LQC
- AQL
- t-Student
- Similar Day (SD)
- LASSO bootstrap
- LASSO point

Market data

Tomasz Serafin (Wrocław, PL)
Prediction bands: the direct approach

- The approach based on forecasted price paths
- Extreme prices at each time point are identified and trajectories containing these values are discarded
The procedure is repeated until $\alpha\%$ of trajectories are removed.
The prediction band with simultaneous coverage probability $1 - \alpha\%$ is formed by linking maximum values of the remaining trajectories.
LASSO-QR-Copula (LQC) procedure

- **Point forecasts**: LASSO → QR → Gaussian Copula → Random quantiles at $t_1$ → Direct
- **Probabilistic forecasts**: LASSO → QR → Multivariate t-Student
- **Path forecasts**: Historical vectors of increments → Historical vectors of errors
- **Starting point for paths**: VWAP at $t_0$ → VWAP at $t_0$ → Direct → Direct → Direct → Naive
- **Prediction band**: LQC → AQL → t-Student → Similar Day (SD) → LASSO bootstrap → LASSO point

Market data

Tomasz Serafin (Wrocław, PL)
Point forecasts

- The VWA price in the $j$-th 15-minute period $t_j$, on day $d$ and hour $h$ is forecasted with LASSO-estimated model.
- Considered variables:
  - Past intraday and day-ahead electricity prices
  - Wind generation and consumption actual values
  - Wind generation and consumption forecasts
Probabilistic & path forecasts

- Quantile regression is used to obtain 99 percentiles of the price in each 15-min interval.
- Trajectories are created by selecting correlated quantiles (time-dependency is captured by the correlation matrix) in subsequent time points.
- The matrix is estimated based on the historical coverage errors of forecasted price quantiles (Pinson et al. (2009)).
LASSO bootstrap procedure

- **Point forecasts**: LASSO
- **Probabilistic forecasts**: QR
- **Path forecasts**: Gaussian Copula
- **Starting point for paths**: Random quantiles at $t_1$
- **Prediction band**: Direct
- **LQC**: AQL
- **AQL**: t-Student
- **Similar Day (SD)**: Historical vectors of increments
- **Historical vectors of errors**: LASSO bootstrap
- **LASSO point**: Naive

Market data

Tomasz Serafin (Wrocław, PL)
LASSO bootstrap procedure

- Time dependency is modeled with historical errors of the point forecasting model.
- Price trajectories are constructed by adding randomly drawn vectors of historical errors to point forecasts from the LASSO model:

\[
\tilde{X}_{d,h,t_j} = \hat{X}_{d,h,t_j} + \varepsilon_{d^*,h,t_j},
\]

where \(\varepsilon_{d^*,h,t_j}\) is the \(j\)-th component of a randomly drawn vector of past errors of the LASSO model.
Prediction bands: the AQL method

Point forecasts:
- LASSO
- QR
- Gaussian Copula
- Random quantiles at $t_1$
- Direct

Probabilistic forecasts:
- Multivariate t-Student
- Historical vectors of increments
- Historical vectors of errors

Path forecasts:
- VWAP at $t_0$

Starting point for paths:
- Direct

Prediction band:
- LQC
- AQL
- Similar Day (SD)
- LASSO bootstrap
- LASSO point

Market data:
Prediction bands: the AQL method

- The approach based on forecasted quantiles
- The quantile line of order $1 - \alpha$:
  \[
  \left( \hat{q}_{d,h,t_1}^{(1-\alpha)}, \ldots, \hat{q}_{d,h,t_{10}}^{(1-\alpha)} \right)
  \]
- We assume that the prediction band with simultaneous coverage probability $1 - \alpha$ is a quantile line of order $1 - \alpha^*$
Prediction bands: the AQL method

- The order of the quantile line, $1 - \alpha^*$, is approximated using the historical coverage of quantile lines from the previous 91 days:

![Graph showing the historical simultaneous coverage probability vs. quantile line order]
Prediction bands: the point forecasts based method
Prediction bands: the point forecasts based method

- This benchmark is solely based on point forecasts from the LASSO model.
- The naive prediction band (not dependent on $\alpha$) is a vector of point forecasts for the consecutive 15-minute periods:

$$B_{d,h,t_i}^{up} = \hat{P}_{d,h,t_i},$$

for all $j = 1, \ldots, 10$
Trading strategies
Real-life market simulation

- A small energy producer
- Sells 1 MWh of electricity in the continuous German intraday market, each hour, each day
Prediction band-based strategy

Prediction bands determine the price of the *limit order* which is placed in the market every 15 minutes.

![Graph showing price trends over time](image-url)
Results

![Graph showing the results of different models: AQL, LASSO bootstrap, LQC, LASSO point, and Naive last. The x-axis represents the simultaneous coverage probability p, ranging from 5% to 95%, and the y-axis represents profit in thousands of EUR, ranging from 176.5 to 182. The graph compares the performance of these models under Gaussian and historical vectors of errors.](#)
Conclusions

- Prediction bands are a great tool to assess the economic value of probabilistic and path forecasts.
- Strategies based on the more complex forecasting approaches result in higher profits for the trading company.
- Trajectory forecasts are a viable alternative to probabilistic forecasts and their use may bring potential gains for the electricity trading company.
Thank you very much!
LASSO-QR-Copula (LQC) procedure

**Point forecasts**
- LASSO
- QR
- Gaussian Copula
- Random quantiles at $t_1$
- Direct
- LQC

**Probabilistic forecasts**
- LASSO
- QR
- Multivariate $t$-Student
- Historical vectors of increments
- Historical vectors of errors

**Path forecasts**
- AQL
- Direct
- VWAP at $t_0$
- Direct
- Similar Day (SD)
- LASSO bootstrap
- LASSO point

**Starting point for paths**
- Naive

**Prediction band**
- Market data
Point forecasts

The model with potential regressors for the VWA price in the $j$-th 15-minute period $t_j$, on day $d$ and hour $h$ is given by:

$$X_{d,h,t_j} = \beta_0 + \sum_{i=4}^{24} \beta_{i-3} I D_{3,d,h-i} + \sum_{i=0}^{24} \beta_{22+i} D A_{d,h-i} +$$

$$+ \sum_{i=0}^{24} \beta_{47+i} \hat{W}_{d,h-i} + \beta_{72} W_{d,h-4} + \beta_{73} W_{d,h-24} +$$

$$+ \sum_{i=0}^{24} \beta_{74+i} \hat{C}_{d,h-i} + \beta_{99} C_{d,h-4} + \beta_{100} C_{d,h-24} +$$

$$+ \beta_{101} \tilde{P}_{d,h},$$
Point forecasts

The model with potential regressors for the VWA price in the $j$-th 15-minute period $t_j$, on day $d$ and hour $h$ is given by:

$$X_{d,h,t_j} = \beta_0 + \sum_{i=4}^{24} \beta_{i-3} ID_{3,d,h-i} + \sum_{i=0}^{24} \beta_{22+i} DA_{d,h-i} +$$

$$+ \sum_{i=0}^{24} \beta_{47+i} \hat{W}_{d,h-i} + \beta_{72} W_{d,h-4} + \beta_{73} W_{d,h-24} +$$

$$+ \sum_{i=0}^{24} \beta_{74+i} \hat{C}_{d,h-i} + \beta_{99} C_{d,h-4} + \beta_{100} C_{d,h-24} +$$

$$+ \beta_{101} \hat{P}_{d,h},$$
Point forecasts

The model with potential regressors for the VWA price in the $j$-th 15-minute period $t_j$, on day $d$ and hour $h$ is given by:

$$X_{d,h,t_j} = \beta_0 + \sum_{i=4}^{24} \beta_{i-3} ID_{3,d,h-i} + \sum_{i=0}^{24} \beta_{22+i} DA_{d,h-i} +$$

$$+ \sum_{i=0}^{24} \beta_{47+i} \hat{W}_{d,h-i} + \beta_{72} W_{d,h-4} + \beta_{73} W_{d,h-24} +$$

$$+ \sum_{i=0}^{24} \beta_{74+i} \hat{C}_{d,h-i} + \beta_{99} C_{d,h-4} + \beta_{100} C_{d,h-24} +$$

$$+ \beta_{101} \tilde{P}_{d,h},$$
Point forecasts

The model with potential regressors for the VWA price in the $j$-th 15-minute period $t_j$, on day $d$ and hour $h$ is given by:

\[
X_{d,h,t_j} = \beta_0 + \sum_{i=4}^{24} \beta_{i-3} ID_{3,d,h-i} + \sum_{i=0}^{24} \beta_{22+i} DA_{d,h-i} + \\
\sum_{i=0}^{24} \beta_{47+i} \hat{W}_{d,h-i} + \beta_{72} W_{d,h-4} + \beta_{73} W_{d,h-24} + \\
\sum_{i=0}^{24} \beta_{74+i} \hat{C}_{d,h-i} + \beta_{99} C_{d,h-4} + \beta_{100} C_{d,h-24} + \\
\beta_{101} \tilde{P}_{d,h},
\]

Tomasz Serafin (Wrocław, PL)
Probabilistic forecasts

- Quantile regression is used to obtain 99 percentiles of the price in each 15-min interval.
- The value of quantile $\alpha$ is given by:

$$\hat{q}_{d,h,t_j}^{(\alpha)} = w_{\alpha,t_j} \hat{X}_{d,h,t_j},$$

where $w_{\alpha}$ is a vector of weights for quantile $\alpha$, estimated by minimizing the pinball score:

$$\text{Pinball}(\hat{q}_{t}^{(\alpha)}, X_t, \alpha) = \begin{cases} (1 - \alpha)(\hat{q}_{t}^{(\alpha)} - X_t) & \text{for } X_t < \hat{q}_{t}^{(\alpha)}, \\ \alpha(X_t - \hat{q}_{t}^{(\alpha)}) & \text{for } X_t \geq \hat{q}_{t}^{(\alpha)}, \end{cases}$$
Gaussian Copula

- Trajectories are created by selecting correlated quantiles in subsequent time points
- Time-dependencies are captured by the correlation matrix
- The matrix is estimated based on the historical coverage errors of forecasted price quantiles (Pinson et al. (2009))
Additional strategies

- **Naive\textsubscript{first}**: The electricity is sold for the market price 3 hours and 15 minutes before the delivery
- **Naive\textsubscript{last}**: The electricity is sold at the last price in the market
- **Naive\textsubscript{avg}**: The electricity is sold in 10 portions of 0.1 MW in each sub-period

Performance of a strategy depends on the choice of the simultaneous coverage probability ($p$) of prediction bands.

We can use a rolling 91-day window for the automated selection of the optimal value of $p$. 
Evaluation of strategies

- Total profit from selling 1 MWh of electricity every hour \( h \) each day \( d \) of the \( N = 200 \) out-of-sample period:

\[
\text{Profit} = \sum_{d=1}^{N} \sum_{h=1}^{24} G_{d,h}
\]

- 90%, 95% and 99% Value-at-Risk of daily profits
Results
Results

- **Maximum** possible profit (crystal ball):
  - 196 650 EUR

- **Minimal** possible profit:
  - 157 590 EUR

- **LQC** method:
  - 181 950 EUR (ca. 63% of possible gains from forecasting)

- **Naive** methods:
  - from 176 135 to 177 265 EUR (ca. 48 to 51% of possible gains from forecasting)
## Results

<table>
<thead>
<tr>
<th></th>
<th>90% VaR</th>
<th>95% VaR</th>
<th>99% VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-Student</td>
<td>622.10</td>
<td>349.64</td>
<td>-231.38</td>
</tr>
<tr>
<td>AQL</td>
<td>645.05</td>
<td>409.16</td>
<td>-143.96</td>
</tr>
<tr>
<td>SD</td>
<td>622.10</td>
<td>345.28</td>
<td>-240.07</td>
</tr>
<tr>
<td>LQC</td>
<td>643.16</td>
<td>425.05</td>
<td>-116.08</td>
</tr>
<tr>
<td>Naive&lt;sub&gt;first&lt;/sub&gt;</td>
<td>631.85</td>
<td>392.90</td>
<td>-143.97</td>
</tr>
<tr>
<td>Naive&lt;sub&gt;last&lt;/sub&gt;</td>
<td>634.70</td>
<td>418.18</td>
<td>-232.83</td>
</tr>
<tr>
<td>Naive&lt;sub&gt;avg&lt;/sub&gt;</td>
<td>629.47</td>
<td>407.28</td>
<td>-171.44</td>
</tr>
<tr>
<td>LASSO bootstrap</td>
<td>650.68</td>
<td>405.96</td>
<td>-139.36</td>
</tr>
<tr>
<td>LASSO point</td>
<td>627.52</td>
<td>385.08</td>
<td>-147.27</td>
</tr>
</tbody>
</table>